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# SIMULATION OF THE FLOW OF VISCOUS-PLASTIC BAROTROPIC COMPRESSIBLE MATERIAL IN CHANNELS OF COMPLEX GEOMETRY

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## Abstract

**In this article we observed the modeling the movement of viscous-plastic material, compressed with indicators that depend on the pressure in the channels of complex geometry of technological equipment. As the material we chose the Bingham fluids with two constant parameters - viscosity and pressure threshold. We consider the flow in the flat channel. The motion of the boundaries is assumed to be purely longitudinal, and the flow field is not longitudinal. The transverse component of the velocity appears due to the dependence of the parameters of the rheological model on pressure. Flow simulation was carried out in two stages. At the first stage, we considered the rheological model of the flow of Bingham material, but without compression. In the second stage, we studied the influence of barotropic compression factor. The superposition method was used when considering the flow model. According to the results of the simulation, we obtained a flow model of nonclassical Bingham material, which can be extended to materials that are compressed and satisfy the conditions of barotropicity. Within the framework of the proposed model, we obtained the equations of zero and the second approximation to determine the characteristics of the Bingham flow in a flat channel. The obtained formulas allow to determine the velocity of the quasi-solid core, which is determined by the degree of deviation of the viscosity and shear threshold, and the average value of the inverse viscosity at the same pressure range. It is established that for long channels in the zero approximation the compressibility does not have a significant effect on the flow, but has an effect only in the second approximation, while the average pressure gradient remains a finite value, ie the pressure difference at the channel ends becomes infinite. For barotropic compressible material, this means that if the pressure at one end of the channel becomes infinite, the density of the material at this end must also become infinite. Thus, when the value of pressure tends to infinity, the value of density remains a finite value. Mathematical description of the flow of viscous-plastic baratropic fluid in the channels of complex geometry of technological equipment allows to establish the dependence of viscosity and limit value of shear stress on pressure with the maximum generalization of parameters.**

*Keywords:* fluid; viscous-plastic; barotropic, flow; model; channel.

# МОДЕЛЮВАННЯ ТЕЧІЇ В'ЯЗКО-ПЛАСТИЧНОГО БАРОТРОПНОГО МАТЕРІАЛУ, ЩО СТИСКАЄТЬСЯ В КАНАЛАХ СКЛАДНОЇ ГЕОМЕТРІЇ

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## Анотація

**Розглянуто особливості моделювання руху в'язко-пластичного матеріалу, що стискається з показниками, залежними від тиску в каналах складної геометрії технологічного обладнання. Як матеріал обрано бінгамовські рідини з двома постійними параметрами – в'язкістю та порогом зрушення, які залежать від тиску. Розглядається течія в пласкому каналі. Рух границь передбачається чисто поздовжнім, а поле течії при цьому є поперечним. Моделювання течії здійснювалось в два етапи. На першому етапі розглядається реологічна модель течії бінгамівського матеріалу, але без стискання. На другому етапі проводилось вивчення впливу фактора баротропного стискання. При розгляді моделі течії використано метод суперпозицій. У рамках запропонованої моделі отримані рівняння нульового та другого наближення для визначення характеристик бінгамівської течії в пласкому каналі. Стискаємість в'язко-пластичного матеріалу показана у величинах тиску та різниці швидкостей на границях каналу. Математичний опис течії в'язко-пластичної баротропної рідини у каналах складної геометрії дозволяє з максимальною узагальненістю параметрів встановити залежність в'язкості і граничного значення напруження зрушення від тиску.**

*Ключові слова:* рідина; в'язко-пластична; баратропна; течія; модель; канал.

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**Introduction**

The majority of processes of chemical and food technologies take place in conditions of barotropicity. Local pressure values that occur during the flow of viscoplastic materials determine the level of mechanophysical or mechanochemical transformations. The dependence of viscosity on pressure at the macro level reflects intermolecular interactions, and the dependence of the shear threshold on pressure in Bingham material shows a change in the degree of consolidation of material elements [1].

The study of the movement of Bingham materials provides the necessary information in order to qualitatively and scientifically organize the various processes of material processing [2; 3]. The motion of Bingham materials, like the motion of any other material with non-Newtonian rheology, equally depends on the equation of state and on the boundary conditions in which motion occurs [4]. Boundary conditions ensure the uniqueness of the solution and its connection with the shape of the motion area, and the equation of state determines the general form of the solution [5].

**Analysis of recent research and publications.**

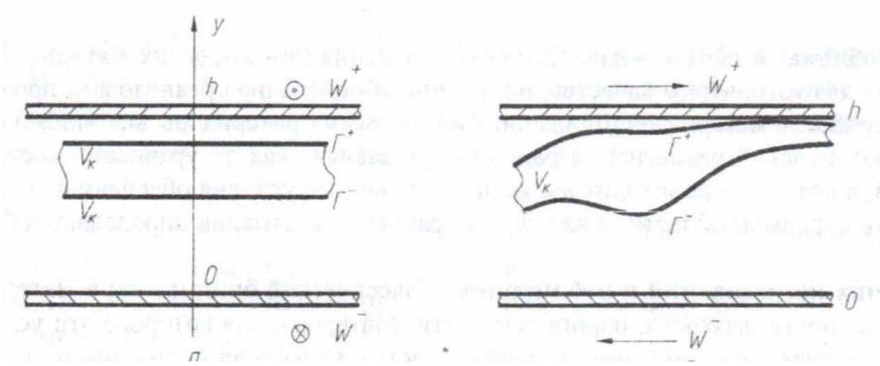
The classic Bingham material implies the constancy of density, viscosity, and fluidity threshold. A material which does not satisfy these conditions is considered nonclassical. The dependence of the characteristics of the equation of state on the arguments is a power or rational function of the invariants of the deformation rate tensor [6–8]. The introduction of pressure into the equation of state as an argument is much less common, but it is quite consistent with the tradition, meaning that the classical equations of state, being unable to correspond to real materials, requires a non-classical extension. One of these types of expansion is the introduction of pressure dependence into the equation of state through viscosity and yield threshold. In combination with the barotropic conditions, a model is obtained that covers many real materials of food and chemical technologies. The materials used there are characterized by a wide variety of properties, especially in the food industry. Often there are mixtures of materials of different nature, acting as binders, fillers, thickeners, dyes, flavors, stabilizers, etc. [9; 10]. In terms of their function in the finished product, each of the materials listed here makes a certain contribution to the equation of state. For low-viscosity materials, the role of the components in the mixture plays a lesser role than for high-viscosity materials, being limited to convective mass transfer [11; 12]. For highly viscous materials, the components in the mixture change the equation of state, predetermining the general properties of the flow. The local values of shear stress pressure arising during the flow determine the level of mechanophysical and mechanochemical effects on the material components. The dependence of viscosity on pressure reflects intermolecular interactions at the macro level, and the pressure dependence of the fluidity threshold in a Bingham material usually reflects a variable degree of consolidation of the structural elements of the material.

Features of the movement of highly viscous materials, including those of Bingham, are the main factor in the design of worm (screw) machines. In these machines, the main element of the working chamber is a trapezoidal channel, the sides of which can be straight or curved segments. The cross sections of such channels are usually approximated by a rectangle [13–15]. This approximation raises the question of which rectangle is the best. The isoperimetric approximation is usually used [16]. The entire working chamber of the worm machine is a set of channels with different cross sections, which are replaced by a set of rectangular channels of the same length. The walls of the channels are formed by the inner surface of the machine body and the outer surfaces of the worms (screws) or worm elements (if the worm is composite) [14–17]. The movement of the worm relative to the body means the movement of various walls of the channel in different directions. Despite the numerical differences, the speed of movement of all the walls of the channel is proportional to the speed of rotation of the worm, and depends on the angle of elevation of its helix [16; 17]. To increase the pressure and shear stresses in the material located in the working chamber of the machine, the channels of which it consists are made shallow [15–17]. In such channels, the height of the rectangle lying in the cross section can be considered small in comparison with the width, so that the channel can be considered flat.

**Results of the research and their discussion**

We consider the flow of barotropic compressive material in a flat channel, viscosity and shear threshold, which depend on pressure. The motion of the boundaries is assumed to be purely longitudinal, and the flow field is not longitudinal. The transverse component of the velocity appears due to the dependence of the parameters of the rheological model on the pressure. Flow simulation is carried out in two stages. At the first stage, we consider the rheological model of the flow of Bingham material, but without compression. In the second stage we study the influence of barotropic compression factor. When considering the flow model, we used the method of superpositions [18; 19].

The flow in the channel in cross and longitudinal sections is shown in Fig. 1.




a b

**Fig. 1. Flow diagram of the material in the channel: a – cross section; b – longitudinal section**

The equations of flow in stresses are as follows:

$$\frac{\partial P}{\partial z}=\frac{\partial \tau_{zz}}{\partial z}+\frac{\partial \tau_{zy}}{\partial y};\ \upsilon_z=\upsilon_z(z,y);\qquad \tau_{ik}=(\mu+\frac{\tau}{\sqrt{I_2}})(\frac{\partial \upsilon_i}{\partial \upsilon_k}+\frac{\partial \upsilon_k}{\partial \upsilon_i});$$

$$\frac{\partial \upsilon_z}{\partial z}+\frac{\partial \upsilon_y}{\partial y}=0;\ \upsilon_y=\upsilon_y(z,y);\quad \mu=\mu(P); I_2=\sum\nolimits_{ik}(\frac{\partial \upsilon_i}{\partial \upsilon_k}+\frac{\partial \upsilon_k}{\partial \upsilon_i})^2 \qquad (1)$$

$$\tau^2=2\tau_{zz}^2+2\tau_{zy}^2;\quad \tau=\tau(P).$$

where $\tau_{zz}$ and $\tau_{zy}$ – components of the shear stress tensor in the material; $P$ – pressure in the material; $z$ and $y$ – longitudinal and transverse coordinates in the channel, respectively; $\upsilon_z$ and $\upsilon_y$ – longitudinal and transverse components of the velocity of materials in the channel, respectively; $\mu$ and $\tau$ – viscosity and fluidity threshold of the material, respectively.

Based on the method of superpositions, which is described in [20–22], relation (1) leads to the problem of flow with longitudinal velocity $\upsilon_z$, which depends on the variables $z$ and $y$, which is defined as a trinomial for the variable $y$, whose coefficients depend only from the longitudinal coordinate. This idea is a factorization of the decision and is an approximation to the exact solution. The solution in this form corresponds to the boundary conditions of the flow, resulting in two equations regarding the width of the solid core and its boundaries. If the flow of Bingham material occurs with constant viscosity values and a shear threshold, the model equations immediately obtain the values of the boundary of the core and its velocity. In this case, when the viscosity and the shear threshold depend on the pressure, the final equations are not algebraic but mixed, ie differential algebraic. The equations also allow us to find the dependence of the pressure on the longitudinal coordinate, after which we can obtain the dependences for the boundaries of the solid core and determine the speed of its motion.

The essence of the proposed approach is to express the component of the stress tensor $\tau_{zz}$ through the component $\tau_{zy}$ both for the equilibrium equation in stresses and for the condition at the core boundaries [21; 22]. Thus, the component $\tau_{zz}$ is expressed through the component $\tau_{zy}$ as follows:

$$\frac{\tau_{zz}}{\tau_{zy}}=F_L\frac{\frac{\partial \upsilon_z^{\pm}}{\partial \zeta_L}.(1\pm\gamma^{\pm})}{\frac{\partial \upsilon_z^{\pm}}{\partial \xi}-F_L^2\frac{\partial}{\partial \zeta_L}\int\frac{\partial \upsilon_z^{\pm}}{\partial \zeta_L}d\xi};$$

$$\zeta = \frac{z}{L}; \qquad 0 \le \zeta_L \le 1;$$
$$\xi = \frac{y}{h}; \qquad 0 \le \xi_L \le 1; \tag{2}$$
$$\mathrm{F}_L = \frac{h}{L}; \qquad \gamma^{\pm} = \frac{\Gamma^{\pm}}{h},$$

where $L$ – the length of the rectangular channel; $h$ – the height of the rectangle in the cross section of the channel; $\Gamma\pm$ – coordinates of the boundaries of the solid core, signs "+" and "-" indicate the values of flow velocities located between the upper boundary of the channel and the core (plus), and located between the lower boundary of the channel and the core (minus).

If for the transverse derivative of the longitudinal flow velocity we use an estimate: $\frac{\partial \upsilon_z^{\pm}}{\partial \xi} \sim \left(w^{\pm} - \upsilon_k\right)\left(1 \mp \gamma^{\pm}\right)$, in which $w^{\pm}$ – the longitudinal velocities of the channel boundaries, and $\upsilon_k$ – velocity of the solid core, then the relation $\frac{\tau_{zz}}{\tau_{zy}}$ can be presented as follows:

$$\frac{\tau_{zz}}{\tau_{zy}} = \mathrm{F}_L \rho^{\pm} \cdot (1 \pm \gamma), \quad \rho^{\pm} = \frac{\dfrac{\partial \upsilon_2^{\pm}}{\partial \zeta_L}}{w^{\pm} - \upsilon_k - \mathrm{F}_L^2 \dfrac{\partial}{\partial \zeta_L} \displaystyle\int \frac{\partial \upsilon_z^{\pm}}{\partial \zeta_L} d\xi}. \tag{3}$$

It should be noted that the notation of the flow equation in stresses by dimensionless coordinates $\zeta_L$ and $\xi$ leads to the fact that the derivative of the pressure along the longitudinal coordinate acquires the factor $\mathrm{F}_L$, which due to the geometry of the channels is always much higher than one. In the extreme case, when $\mathrm{F}_L \to 0$ the driving force of the process, ie the longitudinal pressure gradient disappears. But it should be noted that with an unlimited increase in the length of the channel, the pressure difference at its ends increases so that the pressure gradient remains a finite value. To show this fact, it is necessary to use dimensionless pressure $\overline{P}$. The pressure $P$ is associated with dimensionless pressure $\overline{P}$ by multiplier $P^*$ in such a way, that the pressure difference at the ends of the channels is equal $P^*$; and $P$ $\mathrm{F}_L \sim 1$. Then, in fact the multiplier $\mathrm{F}_L$ is not present ant the pressure gradient $\overline{P}$; and the boundary transition $\mathrm{F}_L \to 0$ can be performed without losing the driving force of the flow. All of the above is equivalent, the operation of dividing the components of the stress tensor $\tau_{zz}$ and $\tau_{yz}$ by the amount of pressure change in the transverse direction at a distance of the channel width, which equals to $\left(P_к - P_н\right)h/L$. In order not to enter new symbols, instead of $\bar{\tau}_{zz}$ and $\bar{\tau}_{zy}$ we adopted the dimensionless components $\bar{\tau}_{zz} / \left(\left(P_к - P_н\right)h/L\right)$ та $\bar{\tau}_{zy} / \left(\left(P_к - P_н\right)h/L\right)$.

In dimensionless coordinates, the derivatives of the component $\bar{\tau}_{zy}$ in the directions have the same order: $\frac{d\bar{\tau}_{zy}}{d\zeta_L} = \frac{d\bar{\tau}_{zy}}{d\xi}$. Taking into account this ratio and the above, the following equations and boundary conditions for the value of the longitudinal velocity $\upsilon_z^{\pm}(z, y)$ can be written as follows:

$$\frac{d\overline{P}}{d\zeta_L} \cdot \frac{1}{1 + \mathrm{F}_L^2 P^{\pm}(1 \mp \gamma^{\pm})} = \frac{d\bar{\tau}_{zy}}{d\xi}; \qquad \bar{\tau}_{zy} = \mu\left(\frac{\partial \upsilon_z^{\pm}}{\partial y} + \frac{\partial \upsilon_y^{\pm}}{\partial z}\right);$$

$$\bar{\tau}_{zy}(\gamma^{\pm}) = \pm \frac{\dfrac{\bar{\tau}}{\sqrt{2}}}{\sqrt{1 + \mathrm{F}_L^2 (P^{\pm})^2 \cdot (1 \mp \gamma^{\mp})^2}}; \tag{4}$$

$$\upsilon_z^{+}(h) = w^{+}; \qquad \upsilon_z^{-}(-h) = w^{-};$$
$$\upsilon_z^{+}(\Gamma^{+}) = v_k, \qquad \upsilon_z^{-}(\Gamma^{-}) = \upsilon_k.$$

After integrating equation (4), we obtain the following expression for velocity $\upsilon_z^{\pm}$:

$$\upsilon_z^{\pm}(\zeta_L,\xi)=\frac{h}{2\mu}\frac{d\bar{P}}{d\zeta_L}\frac{\xi^z}{1+F_L^z P^{\pm}(1\mp\gamma^{\pm})}+c_1\xi+(F_L^z\cdot\frac{\partial}{\partial\zeta_L}\int\frac{d\upsilon_z^{\pm}}{d\zeta_L}d)\xi+c_2^{\pm}, \quad (5)$$

where $c_1$ and $c_2^{+}$ – constants that need to be found using boundary conditions.

Substitution in equation (5) values $\xi=\pm1$, and $\gamma^{\pm}$ and anso the considtion (1) at the boundaries of the solid core leads to the following system of equations for $c_2^{\pm}$:

$$\frac{h}{2\mu}\frac{d\bar{P}}{d\zeta_L}\frac{1}{1+\mathrm{F}_L^2 P^{+}(1-\gamma^{+})}+\mathrm{F}_L^2\frac{\mu}{h}\Delta^{+}+c_1+c_2^{-}=w^{+};$$

$$\frac{h}{2\mu}\frac{d\bar{P}}{d\zeta_L}\frac{1}{1+\mathrm{F}_L^2 P^{-}(1+\gamma^{-})}-\mathrm{F}_L^2\frac{\mu}{h}\Delta^{-}-c_1+c_2^{-}=w^{-};$$

$$\frac{h}{2\mu}\frac{d\bar{P}}{d\zeta_L}\frac{(\gamma^{+})^2}{1+\mathrm{F}_L^2 P^{+}(1+\gamma^{+})}+\mathrm{F}_L^2\frac{\mu}{h}\Delta^{+}\gamma^{+}-c_1\gamma^{+}+c_2^{-}=$$

$$=\frac{h}{2\mu}\frac{d\bar{P}}{d\zeta_L}\frac{(\gamma^{-})^2}{1+\mathrm{F}_L^2 P^{-}(1+\gamma^{-})}+\mathrm{F}_L^2;$$

$$\Delta^{-}\gamma^{-}+c_1\gamma^{-}+c_2^{-};$$

$$\frac{d\bar{P}}{d\zeta_L}\cdot\frac{\gamma^{+}}{1+F_L^2P^{+}(1-\gamma^{+})}+c_1=\frac{\bar{\tau}\sqrt{2}}{\sqrt{1+F_L^2(P^{+})^2(1-\gamma^{+})^2}}; \quad (6)$$

$$\frac{d\bar{P}}{d\zeta_L}\cdot\frac{\gamma^{-}}{1+F_L^2P^{-}(1+\gamma^{-})}+c_1=\frac{\bar{\tau}\sqrt{2}}{\sqrt{1+F_L^2(P^{-})^2(1+\gamma^{-})^2}};$$

$$\Delta\pm\equiv\frac{\partial}{\partial\zeta_L}\int\frac{\partial\upsilon_2^{\pm}}{\partial\zeta_L}d\xi.$$

From the last two equations, the expressions for $c_1$ are substituted into the first two equations, after which these constants are excluded. The third equation of the system (6) and the difference of the last two equations leads to two equations to determine the pressure and limits of the core:

$$\frac{(1-\gamma^{+})^2}{1+\mathrm{F}_L^2P^{+}(1+\gamma^{+})}-\frac{(1+\gamma^{-})^2}{1+\mathrm{F}_L^2P^{-}(1+\gamma^{-})}=\frac{2\mu(w^{+}-w^{-})}{hd\bar{P}/d\zeta_L}+\mathrm{F}_L^2\frac{2\mu}{hd\bar{P}/d\zeta_L}\left[\Delta^{+}(1-\gamma^{+})+\Delta^{-}(1+\gamma^{-})\right]$$

$$\frac{\gamma^{+}}{1+\mathrm{F}_L^2P^{+}(1-\gamma^{+})}-\frac{\gamma^{-}}{1+\mathrm{F}_L^2P^{-}(1+\gamma^{-})}=\frac{\bar{\tau}/\sqrt{2}}{d\bar{P}/d\zeta_L}\cdot\left[\frac{1}{1+\mathrm{F}_L^2(P^{+})^2(1-\gamma^{+})^2}+\frac{1}{1+\mathrm{F}_L^2(P^{-})^2(1-\gamma^{-})^2}\right] \quad (7)$$

$$\upsilon_k=\frac{w^{+}+w^{-}}{2}-\frac{h}{4\mu}\frac{d\bar{P}}{d\zeta_L}\left[\frac{(1-\gamma^{+})^2}{1+\mathrm{F}_L^2P^{+}(1-\gamma^{+})}+\frac{(1-\gamma^{-})^2}{1+\mathrm{F}\frac{2}{L}P^{-}(1-\gamma^{-})}\right]+\mathrm{F}_L^2\frac{1}{2}\left[\Delta^{+}(1-\gamma^{+})+\Delta^{-}(1+\gamma^{-})\right].$$

As seen, the expression for determining the velocity of a solid core $\upsilon_k$ is a first-order equation for pressure $\bar{P}(\zeta_L)$, and the pressure itself must satisfy the boundary conditions at the ends of the channel: $\bar{P}(\zeta_l=0)=\bar{P}_н$; $\bar{P}(\zeta_l=1)=\bar{P}_к$. To do this, there is one integration constant, and the value $\upsilon_k$ is the second integration constant. The first two equations in (7) are used to find the core boundaries $\gamma\pm$ for a material in which $\mu$ and $\tau$ are constant values, $\frac{d\bar{P}}{d\zeta_L}=const$. Thus we obtain the equation for determining the value $\upsilon_k$ while the boundaries $\gamma\pm$ of the core become constant, and $\Delta^{\pm}\equiv0$.

Based on the above, the flow model of nonclassical Bingham material, which is described by the system of equations (7) can be extended to materials that are compressed and satisfy the conditions of barotropicity. Barotropic material differs from non-compressible material in the general form of the equation of conservation of matter and the type of diagonal components of the strain rate tensor [23; 24]. Thus, the equation of state and conservation of the amount of matter can be written as follows:

$$\sigma_{ik} = -P\sigma_{ik} + \left(\mu + \frac{\tau}{\sqrt{I_2}}\right)\left(\frac{\partial \upsilon_i}{\partial \mathrm{F}_k} + \frac{\partial \upsilon_k}{\partial \mathrm{F}_i} - \frac{2}{3}\frac{\partial \upsilon_m}{\partial \mathrm{F}_m}\sigma_{ik}\right); \quad (8)$$

$$\frac{\partial}{\partial z}\left(\rho\upsilon_2\right) + \frac{\partial}{\partial y}\left(\rho\upsilon_y\right) = 0:$$

$$\mathrm{I}_2 = \sum_{i,k}\left(\frac{\partial \upsilon_i}{\partial F_k} + \frac{\partial \upsilon_k}{\partial F_i} - \frac{2}{3}\frac{\partial \upsilon_m}{\partial F_m}\sigma^2_{ik}\right), \quad (9)$$

where $\sigma_{i\kappa}$ – components of the full stress tensor in the material.

It follows that expressions and equations (3) - (7) retain their general form, and the representations for the values $\rho^{\pm}$ and $\Delta^{\pm}$ change as follows:

$$P^{\pm} = \frac{1}{3}\cdot\frac{\left[\frac{\partial \upsilon_z^{\pm}}{\partial \zeta_L} + \frac{2\partial}{\partial \xi}\left(\frac{1}{\rho}\int\frac{\partial \rho\upsilon_z^{\pm}}{\partial \zeta_L}d\xi\right)\right]}{w^{\pm} - \upsilon_k - \mathrm{F}_L^2\frac{\partial}{\partial \zeta_L}\left(\frac{1}{P}\int\frac{\partial \rho\upsilon_2^{\pm}}{\partial \zeta_L}d\xi\right)\cdot\left(1\mp\gamma^{\pm}\right)};$$

$$\frac{\partial \upsilon_z}{\partial z} + \frac{\partial \upsilon_y}{\partial y} == \frac{1}{\rho}\frac{dP}{dP}\left(\frac{dP}{dP}\upsilon_z + \frac{dP}{dy}\upsilon_y\right); \quad (10)$$

$$\Delta^{\pm} = \frac{\partial}{\partial \zeta_L}\left(\frac{1}{P}\int\frac{\partial \rho\upsilon_z}{\partial \zeta_L}d\xi\right).$$

Thus, equations (9) with formulas (10) form a model of compressible barotropic Bingham material with a pressure-dependent viscosity and shear threshold.

It should be noted that obtaining the equations of this model is based on two basic elements. The first of them is that the derivatives $\frac{\partial \upsilon_z^{\pm}}{\partial \xi}$ were qualitatively evaluated, and the essence of the second element is that in dimensionless variables the longitudinal and transverse derivative components $\tau_{zy}$ have the same order. Adopting the above assumptions, the model takes the form of equations (9).

Further development of the model can be done in two directions. In the first direction, for the values $\rho^{\pm}$ and $\Delta^{\pm}$ a priori estimates are made using the boundary conditions on the walls of the channel and at its ends. Then, as already noted, equations (9) are solved with respect to the pressure $P(z)$ and the core boundaries $\gamma^{\pm}(z)$, and for the latter the equations are algebraic. The other direction, the values $\rho^{\pm}$ and $\Delta^{\pm}$ are considered functions of the coordinate $z(\zeta_L)$. To determine them, it is necessary to use expression (6) the same way as to determine the speed $\upsilon_z^{\pm}$. In this case, given that the velocity $\upsilon_z^{\pm}$ depends on the values $\gamma^{\pm}$, and $\rho^{\pm}$ and $\Delta^{\pm}$ contain derivatives on *z*, to determine the boundaries of the core $\gamma^{\pm}(z)$ we obtain nonlinear differential equations of the second order. For an unambiguous possibility of solving these equations, it is necessary to set conditions at the ends of the channel: $\gamma^{\pm}(\zeta_L = 0) = \gamma_0^{\pm}$; $\gamma^{\pm}(\zeta_L = 1) = \gamma_L^{\pm}$. Thus, equations (9) are transformed into a set of three differential equations – two second-order equations for $\gamma^{\pm}$, and one first-order differential equation for pressure, in which the value of the constant velocity of the core $\upsilon_k$ is absent.

Next, we considered some solutions of the flow model in the approximation $\mathrm{F}_L = 0$, ie, regardless of the choice of one of the two described directions, and we derive first-order equations for the smallness of the parameter $\mathrm{F}^2_L$

within the above mentioned first direction of the study.

Returning to equation (7), it should be noted that it includes a small parameter $F_L$, which is quite small for the channels that form the working chamber of the worm machine [25]. Therefore, it is expedient to consider the solution of these equations in the approximation $F_L = 0$. In this case, for the derivative $\frac{d\bar{P}}{d\zeta_L}$ the following first-order differential equation is obtained:

$$\left(-\upsilon_k+\frac{w^++w^-}{2}\right)\cdot\frac{4\mu}{h}=\frac{d\bar{P}}{d\zeta_L}\left\{\left[1-\frac{\bar{\tau}}{\frac{d\bar{P}}{d\zeta_L}}-\frac{\mu\cdot\frac{w^++w^-}{2h\frac{d\bar{P}}{d\zeta_L}}}{\frac{\tau}{\frac{d\bar{P}}{d\zeta_L}}-1}\right]^2+\left[1-\frac{\bar{\tau}}{\frac{d\bar{P}}{d\zeta_L}}+\frac{\frac{\mu w^++w^-}{2h\frac{d\bar{P}}{d\zeta_L}}}{\frac{\tau}{\frac{d\bar{P}}{d\zeta_L}}-1}\right]^2\right\}-$$

$$-\frac{2h}{\mu}F_L^2\left[\Delta^+\left(1-\gamma^+\right)+\Delta^-\left(1-\gamma^-\right)\right]. \qquad (11)$$

When solving this equation, we must first consider a flow with fixed boundaries, the driving force of which is the pressure gradient. In this flow $w^+ = w^- = 0$. Based on this, to determine the value of the pressure gradient $\frac{d\bar{P}}{d\zeta_L}$ at fixed boundaries, we obtain the quadratic equation of the following form:

$$\frac{d\bar{P}}{d\zeta_L}=\left(\frac{\upsilon_k\mu}{\Delta P_h h}-\tau\right)\pm\sqrt{\left(\frac{\upsilon_k\mu}{\Delta P_h h}-\bar{\tau}\right)^2-\tau^2},\ \mu=\mu(P),\ \bar{\tau}=\bar{\tau}(P), \qquad (12)$$

where $\Delta P_h = P_к - P_н$.

It should be noted that even when the values $\mu$ and $\tau$ have a fairly simple dependence on pressure $\bar{P}$, this equation is not integrated in the quadratures. For each specific type of dependencies $\mu(\bar{P})$ and $\tau(\bar{P})$ we require a numerical solution. However, there is a special case when, on the one hand, it is possible to obtain a clear result of a large community, and on the other hand - with a significant limitation of this community. This is the case when the values $\mu(\bar{P})$ and $\tau(\bar{P})$ are proportional to each other. If the coefficient of proportionality between them is denoted by $\lambda_P$, then for an arbitrary nature of the dependence, for example, $\mu(\bar{P})$, for pressure $\bar{P}$ we obtain the following relationship:

$$\int_{P_0}^{\bar{P}}\frac{d\bar{P}}{\mu(\bar{P})}=\left[\left(\lambda-\frac{\upsilon_k}{\Delta P_h h}\right)+\sqrt{\frac{\upsilon_k}{\Delta P_h h}\left(\frac{\upsilon_k}{2h_{\Delta P_h}}-\lambda\right)}\right]\zeta_L. \qquad (13)$$

Relation (13) is a continuation of expression (12), where the square sign is preceded by a plus sign. The choice of the sign is due to the fact that if in formula (12) $\tau\equiv 0$, $\mu = const$, then for $\upsilon_k$ we obtain the value of the velocity of the viscous Newtonian flow on the axis of the flat channel. Therefore, the choice of the "plus" sign satisfies such correspondence. The value $\upsilon_k$ s determined from the condition that $\bar{P}=\bar{P}_K$ when $\zeta_L=1$. Hence we get the expression to determine the velocity of the core $\upsilon_k$:

$$\upsilon_k=\frac{h}{2}\cdot\frac{\left(F_\mu+\lambda\right)^2}{F_\mu},\ F_\mu\equiv\int_{\bar{P}_0}^{\bar{P}_L}\frac{d\bar{P}}{\mu\left(\bar{P}\right)}. \qquad (14)$$

In the general case, when $w^+\neq w^-\neq 0$. The result of solving relation (11) leads to the following equation:

$$\left(\frac{\mu}{2h}\frac{\left(w^+-w^-\right)}{\Delta P_h}-\frac{2\upsilon_k\cdot\mu}{\Delta P_h h}\right)\left(\frac{d\bar{P}}{d\zeta_L}-\bar{\tau}\right)^2\frac{dP}{d\zeta_L}=2\left\{\left(\frac{dP}{d\zeta_L}\right)^2\left(\frac{\mu}{2h}\frac{\left(w^+-w^-\right)}{\Delta P_h}\right)^2+\left(\frac{d\bar{P}}{d\zeta_L}-\bar{\tau}\right)^4\right\}. \qquad (15)$$

This equation allows only a numerical solution because it includes all degrees $\frac{d\bar{P}}{d\zeta_L}$. By coarsening, this equation can be reduced to the previous one, which belongs to the case $w^+=w^-= 0$. To do this, we must return to relation (11), from which it follows that in the right part we have the

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squares of the terms of which the expressions in square brackets are composed. From here we can see that if $w^+=w^-= 0$ then we obtain equation (12).

Therefore, instead of equation (15) we can write an equation close to it of the following form:

$$\frac{d\bar{P}}{d\zeta_L}=\left[\frac{1}{R}\left(\frac{\upsilon_k\cdot\mu}{h\Delta P_h}-\frac{\mu\left(w^++w^-\right)}{\Delta P_h h}\right)-\tau\right]+\sqrt{\left[\frac{1}{R}\left(\frac{\upsilon_k\cdot\mu}{h\Delta P_h}-\frac{\mu}{h}\frac{\mu\left(w^++w^-\right)}{\Delta P_h}\right)-\tau\right]^2-\tau^2};$$

$$R=1+\left[\frac{\mu_0+\mu_L}{4h}\cdot\frac{L}{\bar{P}_L-\bar{P}_0}\cdot\left(w^+-w^-\right)\right]^2/\left[1-\frac{\left(\tau_L+\tau_0\right)L}{2\left(P_L-P_0\right)}\right]. \qquad (16)$$

Limiting the case when $\tau(P)=\lambda\mu(P)$ for value $\upsilon_k$ we obtain the following formula for determining the velocity of the core:

$$\upsilon_k=\frac{w^++w^-}{2}+\frac{h}{2}\cdot\frac{\left(F_\mu+\lambda\right)^2}{F_\mu}R. \qquad (17)$$

In order to obtain the equation of the first approximation for the value of the small parameter $F_L$ we must return to equations (7). It is necessary to present all the input values in the form of series of $F^2_L$ as this parameter is presented in these equations only in second degree. It is necessary to decompose the pressure $\bar{P}$, viscosity and shear threshold. In the first equation in (7), the term with the values $\Delta^{\pm}$ should be left in the zero approximation due to the presence of the multiplier $F^2_L$. In expansions of all quantities, zero approximations are marked with the index "zero", and the first approximations - with the index "two". Omitting all intermediate transformations due to their cumbersomeness, the final result takes the form of the following linear system of equations to determine the coordinates of the boundaries of the core $\gamma_2^{\pm}$:

$$\gamma_2^+\left(1-\gamma^+\right)+\gamma_2^-\left(1+\gamma_0^-\right)=-\frac{1}{2}P_0^+\left(1-\gamma_0^+\right)^3+\frac{1}{2}P_0^-\left(1+\gamma^-\right)^3+\frac{w^+-w^-}{h}\frac{\mu\left(\bar{P}^0\right)}{\frac{d\bar{P}^0}{d\zeta_L}};$$

$$\left(\frac{\frac{d\bar{P}_2}{d\zeta_L}}{\frac{d\bar{P}^0}{d\zeta_L}}-\frac{1}{\mu(\bar{P}^0)}\frac{d\mu}{d\bar{P}}|_{\bar{P}^0}\cdot\bar{P}_2\right); \qquad (18)$$

$$\gamma_2^+-\gamma_2^-=P_0^+\gamma_0^+\left(1-\gamma_0^+\right)+P_0^-\gamma-\left(1+\gamma_0^-\right)+\frac{\tau\left(\bar{P}^0\right)}{\frac{d\bar{P}^0}{d\zeta_L}}\cdot\left(\frac{\frac{d\bar{P}_2}{d\zeta_L}}{\frac{d\bar{P}_0}{d\zeta_L}}-\frac{1}{\tau\left(\bar{P}^0\right)}\frac{d\tau}{d\bar{P}}|_{\bar{P}^0}\cdot\bar{P}_2\right).$$

where $\bar{P}^0$ – zero approximation for pressure.

This notation is introduced in (18) in order not to be confused with the notation. $\bar{P}^0$ from the boundary conditions for pressure $\bar{P}(\varsigma_L)=P_0$.

The system of equations (18) is not bounded by constraints $\tau = \lambda\mu$ and has a universal form. If the relation between $\tau$ and $\mu$in the form of proportionality takes place, then as $\gamma_0^{\pm}(\zeta_L)$ and $\bar{P}_0(\zeta_L)$ we should substitute (17) and (16) as functions in the system of equations (7), which is taken with zero approximation. Decomposition of the expression for the velocity of the core $\upsilon_k$ gives for the second approximation the following expression, which can be considered an equation for the specified value of pressure $\bar{P}_2$:

$$\frac{\upsilon_{k2}-\left(\frac{1}{2}\right)\left[\Delta_0^+\cdot\left(1-\gamma_0^+\right)+\Delta_0^-\cdot\left(1+\gamma_0^-\right)\right]}{\frac{h}{2\mu\left(\bar{P}^0\right)}\frac{d\bar{P}^0}{d\zeta_L}\left[\left(1-\gamma_0^+\right)^3-\left(1+\gamma_0^-\right)^3\right]-(w^+-w^-)}\cdot\frac{d\bar{P}^0}{d\zeta_L}=\frac{d\bar{P}_2}{d\zeta_L}-\frac{d\ln\mu(P^0)}{d\zeta_L}\cdot\bar{P}_2 \qquad (19)$$

This first-order linear equation for $\overline{P}_2$ is integrated in quadratures based on the zero approximation, if known. For the case when $\tau = \lambda\mu$, all values are known; and $P^{(0)}(\zeta_L)$ – by equation (13), then the values $\gamma_0^{\pm}$ are as follows:

$$\gamma_0^{+} = \frac{\frac{\tau}{\sqrt{2}}}{\frac{d\overline{P}^0}{d\zeta_L}} \pm \frac{\mu(\overline{P}^0)(w^{+}-w^{-})}{\frac{2hd\overline{P}^0}{d\zeta_L}} / \left( \frac{\tau(\overline{P}^0)}{\frac{d\overline{P}^0}{d\zeta_L}} - 1 \right). \quad (20)$$

**Conclusion**

Thus, within the framework of the proposed model, the equations of zero and the second approximation are obtained to determine the characteristics of the Bingham flow in a flat channel. The non-limiting $F_L$ parameter assumption was used, given the possible applications to fluid flows in helical channels. Equations (7), (18) and (19) are general and do not depend on hypothesis. Different types of dependence $\mu(P)$ and $\tau(P)$ can be studied only numerically. The hypothesis of proportionality of viscosity and shear threshold in a real situation, when $\mu$ and $\tau$ are arbitrary values, characterizes by the parameter $\lambda$ the degree of their difference from each other in the interval of the pressure axis ($P_0$, $P_L$). The result expressed by formula (14) actually means that the velocity of the quasi-solid core is determined by the degree of deviation of the values $\mu$ and $\tau$, and the average value of the inverse viscosity at the same pressure range. As can be seen from formula (15), the velocity of the solid core depends on both the sum and the difference in velocities at the channel boundaries. The compressibility of the Bingham material is shown in the values $\rho^{\pm}$ and $\Delta^{\pm}$, it can be stated that for long channels in the zero approximation the compressibility has no effect on the flow, and in only the second approximation it does. The explanation for this fact is that it is believed that at $F_L = 0$ the average pressure gradient remains a finite value, ie the pressure difference at the ends of the channel becomes an infinite value. For barotropic compressible material, this means that if the pressure at one end of the channel becomes infinite, the density of the material at this end must also become infinite. That is, the density should increase at least as the value $1/F^2_L$. But then the values $\rho^{\pm}$ and $\Delta^{\pm}$, which include the density $\rho(P)$ also become infinitely large so that their product on the small parameter $F^2_L$ becomes finite; and decomposition by a small parameter in equations (7) cannot be performed. However, since such decompositions have been performed, this indicates that the dependence of the density of the material on the pressure is limited. Thus, when the value of pressure tends to infinity, the value of density remains a finite value. Therefore, when constructing the $\rho(\overline{P}^0)$ included in $\rho^{\pm}$ and $\Delta^{\pm}$ we should use dependencies that satisfy the condition $\lim \rho(\overline{P}^0) \to \rho^{\infty}$, when $(\overline{P}^0) \to \infty$.

Based on the above, the presented model of the flow of barotropic fluid in a flat channel allows to establish the dependence of the viscosity and the limit value of the shear stress on the pressure with the maximum generalization of the parameters. This model allows a number of refinements and complications that may be associated with a more accurate definition of $\rho^{\pm}$ and $\Delta^{\pm}$, as well as various numerical studies of the influence of the dependences $\mu(P)$ and $\tau(P)$ on the characteristics of viscoplastic flow.

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