COUPLE STRESS EFFECT ON FERRO-CONVECTION TRIGGERED BY CHEMICAL REACTION IN A POROUS LAYER WITH SPARSE DISTRIBUTION

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Abstract

The study delves into the impact of couple stress on the commencement of convection in a porous material oriented horizontally. This layer contains a chemically reactive ferromagnetic fluid and experiences bottom heating. The investigation utilizes small perturbation methodology to explore and understand the impact of couple stress on the initiation of convection in this specific system. With the assumption of a non-autocatalytic exothermic reaction, eigenvalues are determined utilizing the Galerkin method. The analysis explores the effects of magnetic and couple stress parameters, as well as the Frank-Kamenetskii number. The observation indicates that the acceleration of the onset of ferroconvection is influenced by both magnetic forces and chemical reactions. Simultaneously, the presence of the couple stress component serves to stabilize the system. Moreover, when the nonlinearity of magnetization is sufficiently pronounced, the destabilization of the fluid layer is observed to be marginal.

Keywords: Ferro-convection; Couple Stress; Chemical Reaction; Porous Media; Galerkin Method; Linear Stability Analysis; Normal Mode Technique.

ВПЛИВ ПАРНОГО НАПРУЖЕННЯ НА ФЕРОКОНВЕКЦІЮ, ІНІЦІЙОВАНУ ХІМІЧНОЮ РЕАКЦІЄЮ В ПОРИСТОМ ШАРИ З РОЗРІЖДЕНИМ РОЗПОДІЛОМ

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Анотація

Дослідження присвячене впливу напруження пари на початок конвекції в пористому матеріалі, орієнтованому горизонтально. Цей шар містить хімічно реактивну феромагнітну рідину і зазнає придонного нагріву. У дослідженні використовується метод малих збурень для вивчення і розуміння впливу напруження пари на початок конвекції в цій специфічній системі. З припущенням про неавтокаталітичну екзотермічну реакцію, власні значення визначаються за допомогою методу Галеркіна. Проаналізовано вплив параметрів магнітних і парних напружень, а також числа Франка-Каменецького. Спостереження показують, що на прискорення початку фероконвекції впливають як магнітні сили, так і хімічні взаємодії. Водночас наявність парної компоненти напружень сприяє стабілізації системи. Причому, коли нелінійність намагніченості достатньо виражена, дестабілізація шару рідини спостерігається незначною.

Ключові слова: Фероконвекція; парні напруження; хімічна реакція; пористі середовища; метод Галеркіна; лінійний аналіз стійкості; метод нормальних режимів.

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Introduction

The utilization of an external magnetic field for the control of ferrofluids justifies their application in various engineering contexts. Researchers are currently concentrating on the mechanism of synovial joint lubrication, which is connected to the utilization of couple-stress fluid. A human joint is a dynamically loaded bearing that is lubricated by synovial fluid and is made of articular cartilage. Normal synovial fluid is a viscous, non-Newtonian fluid that can be clear or yellowish. The enhancement of heat transfer through ferro-convection has led to the emergence of numerous compelling and practical uses for magnetic fluids [1]. A growing interest among researchers is evident in the study of the ferro-convective instability phenomenon, driven by the significant alterations induced by magnetic forces in the critical values [2–8].

Concomitant interests in the realm of convection within reaction-driven porous mediums have been prompted by its application in diverse fields, including the functioning of reactors and the synthesis and oxidation of materials. In one of the studies of a horizontal inert porous layer, it was effectively delved into the intricacies of this matter, considering an exothermic chemical reaction characterized by zeroth-order kinetics. They revealed that the instability induced by the chemical reaction can be primarily attributed to the nonlinear temperature distribution of the fundamental state.

The emergence of couple stress is rooted in the premise that the distribution of momentum and force within a material equates to the mechanical interplay between different sections of the body. Notably, these fluids exhibit distinctive attributes, such as the polar effect and the capability to exhibit high viscosity. The Stokes-developed model for couple-stress fluids finds widespread application in elucidating various phenomena related to suspensions.

The decelerating impact of couple stress on convection initiation contrasts with the accelerated onset facilitated by central infiltration studied in [9; 10] leading to the inference that the stability exchange system experienced heating within the perforated region in the couple stress fluid. Previous observations indicate alterations in instability thresholds when the fluid permeating the porous medium exhibits paired-voltage behaviour. Additionally, assuming the porous medium is subject to minor perpendicular oscillations, Taj et al. [11] investigated the convection dynamics of a chemically reactive fluid in a heated porous medium from below using a modified Darcy model. The presence of couple stresses enhance system stability, and this stabilizing effect remains resilient in the face of counteracting influences from chemical reactions, and vice versa. Exploring the Maxwell-Cattaneo law, George & Thomas [12] studied the repercussions of gravitational modulation and formation during the initial phase of modulation within couple voltages. Their findings demonstrate the potential for enhancing or delaying the convection transfer by regulating various governing factors. In magnetic fluid technologies, controlling convection is crucial yet challenging. Despite extensive research, no study has yet analyzed anisotropic Brinkman ferroconvective instability caused by the interplay of magnetic and couple stresses with a non-autocatalytic exothermic reaction. Using the Darcy-Brinkman model with anisotropic permeability to describe porous media flow, and incorporating thermal anisotropy in the energy equation, this study employs stability analysis with normal modes and the higher-order Galerkin method. M.C Krishna Reddy et al. [13] investigated the heat and mass transfer consequences of unsteady MHD free convection flow via a vertical permeable moving plate with radiation. G Murali et al. [14] investigated the heat and mass transfer effects of an unstable hydromagnetic free convective flow across an infinite vertical plate immersed in a porous media with heat absorption. Deepa Gadipally, etc.al.[15] analyzed the soret and dufour effects on unsteady MHD flow via a semi-infinite vertical porous plate using the finite difference approach. Murali et al.[16] investigated the impact of Soret and Dufour on unstable hydromagnetic free convective fluid flow through an infinite vertical porous plate in the presence of a chemical reaction. Murali et al. [17; 18] provided a finite element solution for an MHD-driven problem solving system. An analysis was carried out on Casson fluid performance on natural convective dissipative couette flow past an infinite vertically inclined plate filled in porous medium with heat transfer, MHD and hall current effects by N. V. N Babu etc.al. [19]. A numerical study was done on Convective MHD Jeffrey Fluid Flow Due to Vertical Plates with Pulsed Fluid Suction by Murali et.al [20].
The analysis highlights the influence of horizontal wave number and couple stress fluid parameters on the fluid layer's horizontal linkage, while the perforated parameters significantly impact the predominant fluid layer [21]. Rudresha et al. [22] conducted a recent study examining how Electric Field Modulation and couple stress influence the initiation of electroconvection in a dielectric fluid layer. Their findings [23–25] indicate that the stabilizing and destabilizing effects are contingent upon the frequency involved. The works mentioned in the literature [26–35] had a significant impact on understanding the nature of the reported work.

This study investigates linear and nonlinear stability theories for convective movement of a reactive solute in viscous incompressible fluid. Analyzing the governing model's thresholds for linear and nonlinear stability helps determine convection type and start, evaluating linear theory's suitability for forecasting convection physics. Comparing thresholds helps evaluate the system's stability.

Understanding convection in ferrofluids is crucial for various applications, including heat transfer enhancement, damping in mechanical systems, and the development of advanced engineering solutions for control and actuation mechanisms. Ongoing research in this field aims to uncover the underlying physics, optimize the material properties, and explore the potential applications in a wide range of industries, including biomedicine, electronics, and robotics. In practical situations, it is often feasible to delay or expedite the initiation of convection by altering influential factors. Despite numerous studies on the impacts of couple stress and chemical reactions, the literature on the interplay between chemical reactions, couple stress, and ferrofluids is notably sparse based on our current understanding. Consequently, this research endeavours to examine the influence of couple stress on a chemically reactive ferrofluid within a porous medium.

**Experimental**

**Mathematical Formulation**

The current problem involves an unbounded, horizontally oriented ferrofluid layer with couple stress, contained within two parallel plates of thickness $d$. The passive porous layer is subjected to cooling from the upper region with a temperature of $T_c$. A zeroth-order reaction could be activated as the temperature experiences slight variations across the entire domain from $T_c$.

The temperature at the lower surface is denoted as $T_h$, where $T_h$ is greater than $T_c$. Gravity is assumed to act vertically downwards in this context (see Figure 1).

![Fig.1. Physical Outline](image)

The pertinent fundamental equations for the chemically reactive couple stress magnetic fluid (CSMF) within a saturated porous medium, considering the Boussinesq approximation and a temperature range below the Curie point, are as follows:

$$\nabla \cdot \tilde{q} = 0$$

$$\rho_0 \left[ \frac{1}{\varepsilon} \frac{\partial q}{\partial t} + \frac{1}{\varepsilon^2} (\tilde{q} \cdot \nabla) \tilde{q} \right] = -\nabla p + \rho \tilde{g} + \nabla \cdot (\tilde{H} \tilde{B}) + \mu \nabla^2 \tilde{q} - \frac{\mu'}{k} \tilde{q} - \mu_c \nabla^4 \tilde{q}$$

$$\varepsilon \left[ \rho_0 C_V H - \mu_0 H \left( \frac{\partial M}{\partial T} \right)_{V,H} \right] \left[ \frac{\partial T}{\partial t} + (\tilde{q} \cdot \nabla) T \right] + (1 - \varepsilon)(\rho_0 C_s) \frac{\partial T}{\partial t}$$

$$+ \mu_0 T \left( \frac{\partial M}{\partial T} \right)_{V,H} \left[ \frac{\partial H}{\partial t} + (\tilde{q} \cdot \nabla) H \right] = KV^2 T + Qe^{\left( 0 \right)}$$

$$\rho = \rho_0 \left[ 1 - \beta (T - T_c) \right]$$

$$M = \frac{H}{H_m} (H, T)$$

$$M = M_0 + \chi (H - H_0) - K_m (T - T_c)$$
where \( \vec{q} \) the fluid velocity, \( \rho \) the fluid density, \( p \) the reduced pressure, \( \vec{g} \) the gravitational acceleration, \( \mu \) and \( \mu' \) effective and dynamic viscosity, \( \mu_c \) the couple-stress viscosity, \( \varepsilon \) the porosity of the porous medium and \( k \) the permeability of the porous medium. \( \vec{H}, \vec{B} \) and \( \vec{M} \) are vectors of magnetic field intensity, magnetic induction and magnetization respectively and \( \mu_0 \) is the magnetic permeability.

The chemical reaction effect is introduced by the term \( Q e^{\frac{-E}{RT}} \) where the product of the reactant concentration, a pre-exponential factor and heat of reaction, denoted as \( Q \); the activation energy, represented by \( E \) and the universal gas constant, denoted as \( R \).

Further, \( T \) is the temperature, \( K \) is the effective thermal diffusivity, \( \beta \) is the thermal expansion coefficient, \( \chi \) is the magnetic susceptibility, \( m_K \) is the pyromagnetic coefficient and \( C_{V,H} \) is the specific heat at constant magnetic field and volume.

The pertinent Maxwells equations are given by

\[
\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0
\]

\[
\vec{B} = \mu_0 (\vec{M} + \vec{A})
\]

The temperature is made dimensionless by setting \( \theta = \frac{T - T_c}{T_r} \), where \( T_r = \frac{RT_c^2}{E} \) with \( \frac{RT_c}{E} << 1 \) and the dimensionless temperature boundary conditions aligned with the objective include

\[
\theta = 0 \quad \text{at} \quad z = 1 \quad \text{and} \quad \theta = \theta_h \quad \text{at} \quad z = 0.
\]

The fluid is quiescent in the initial state and is provided by

\[
\vec{q} = \vec{q}_b(z) = 0, \quad \rho = \rho_b(z), \quad p = p_b(z), \quad \theta = \theta_b(z), \quad \vec{H} = \vec{H}_b(z), \quad \vec{M} = \vec{M}_b(z), \quad \vec{B} = \vec{B}_b(z).
\]

The solution for the no-convection state from equations (1) to (8) is provided by

\[
\frac{dp_b}{dz} = -\rho_b g + B_b \frac{dH_b}{dz}
\]

\[
K \frac{d^2 \theta_b}{dz^2} + \frac{Qe^{\frac{-E}{RT}}}{T_r} C_f e^{\theta_b} = 0
\]

\[
\rho_b = \rho_0 [1 - \beta T_r \theta_b]
\]

\[
\frac{d}{dz} (M_b + H_b) = 0
\]

\[
M_b(z) = M_0 + \chi (H_b - H_0) - K_m T_r \theta_b
\]

Upon solving eqn. (13) and using it in eqn. (14) we get

\[
H_b(z) = H_0 + \frac{K_m T_r \theta_b}{1 + \chi}
\]

\[
M_b(z) = M_0 - \frac{K_m T_r \theta_b}{1 + \chi}
\]

\[
B_b(z) = \mu_0 (M_0 + H_0)
\]

Adopting the methodology of Malashetty et al. [16], the solution of eqn. (11) reads:

\[
\theta_b = \ln \left( \frac{l_1}{2F} \right) + \ln \left[ 1 - \left( \frac{1 - l_2 \exp \left( -\sqrt{l_1} z \right)}{1 + l_2 \exp \left( -\sqrt{l_1} z \right)} \right)^2 \right],
\]

where the constants \( l_1 \) and \( l_2 \) in eqn. (18) are implicitly calculated by
\[
\exp\left(\sqrt{1}\right) \frac{1 - \frac{2F}{l_1}}{1 + \frac{2F}{l_1}} = \frac{1 - \frac{2F \exp(\theta_h)}{l_1}}{1 + \frac{2F \exp(\theta_h)}{l_1}} \quad (19)
\]

\[
l_2 = \exp\left(\sqrt{1}\right) \left[1 - \frac{2F}{l_1}\right]
\left[1 + \frac{2F}{l_1}\right], \quad (20)
\]

where \( F = \frac{Ca^2}{\kappa} \) is the Frank-Kamenetskii number, \( C = \frac{Qe^{RT_f}}{Tr} \) and \( \kappa = \frac{K}{\varepsilon Cf} \).

**Stability analysis.** Employing normal modes in the conventional stability analysis [2; 11; 16; 31], the resulting non-dimensional equations are:

\[
\frac{\sigma}{P_r} \left(D^2 - a^2\right)W + R_a a^2 \theta + D_a^2 \left(D^2 - a^2\right)W - \Lambda \left(D^2 - a^2\right)^2 W
+ \Gamma \left(D^2 - a^2\right)^3 W + Na^2 \left(\frac{d\theta_b}{dz}\right)W = 0
\]

\[
\lambda \sigma \theta - \left(D^2 - a^2\right)\theta + \left(\frac{d\theta_b}{dz}\right)W = 0
\]

\[
D^2 \Phi - M_3 a^2 \Phi - D \theta = 0
\]

where \( \sigma \) the rate of growth, \( \Phi \) the magnetic scalar potential, \( P_r = \frac{\mu_f}{\rho_0 \kappa} \) the Prandtl number, \( \Lambda = \frac{\mu}{\mu_f} \) the Brinkman number, \( D_a = \frac{d}{\sqrt{\kappa}} \) the Darcy number, \( R_a = \frac{\beta \rho_0 g d^3 R_T e^2}{\mu_f \kappa} \) the media Rayleigh number, \( \Gamma = \frac{\mu_c \kappa \mu_f d^2}{\mu_f \kappa (1 + \chi)} \) the couple stress parameter, \( M_3 = \frac{1 + M_0}{H_0 (1 + \chi)} \) the non-buoyancy magnetization parameter, \( N = \frac{\mu_0 K_m^2 T_r^2 d^2}{\mu_f \kappa (1 + \chi)} \) the media magnetic number.

**Stationary Instability.** In light of the fact that oscillatory instability is not present, the set of equations pertaining to stationary instability is as follows:

\[
R_a a^2 \theta + D_a^2 \left(D^2 - a^2\right)W - \Lambda \left(D^2 - a^2\right)^2 W
+ \Gamma \left(D^2 - a^2\right)^3 W + Na^2 \left(\frac{d\theta_b}{dz}\right)W - \Lambda \left(D^2 - a^2\right)^2 W
= 0
\]

\[
\left(D^2 - a^2\right)\theta + \left(\frac{d\theta_b}{dz}\right)W + Fe^\theta \theta = 0
\]

\[
D^2 \Phi - M_3 a^2 \Phi - D \theta = 0
\]

The applicable boundary conditions are

\[
W = D^2 W = D^4 W = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1
\]

\[
\theta = D \Phi = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1
\]
The set of equations (24) through (26) along with the boundary conditions (27) constitutes an eigenvalue problem, with $R_d$ representing the eigenvalue. Due to the variable coefficients in the boundary value problem described by equations (24) through (26), analytical solution becomes challenging. Therefore, an approximate solution is pursued using the Galerkin method [24].

**Results and Discussions**

The present study addresses the issue of chemical reaction-driven ferroconvective instability, taking into account the influence of the couple stress effect in a Brinkman type. Due to the absence of oscillatory instability and the impracticality of obtaining an analytical solution, the Galerkin technique is utilized to compute the corresponding numerical solution. Figure 2 displays the temperature profiles for $\theta_b$ and $F$ during the quiescent state. The fluid’s reaction is self-sustaining, and the lower boundary's nature shifts to adiabatic pasta specific value of $F$ [16]. Our analysis focuses solely on $F$ below this critical value. The critical threshold of $F$ is determined to be 0.878455 for $\theta_b = 1.19$. It is evident that an increase in $F$ leads to heightened imbalance and deviation from linearity in the temperature profiles during the inactive state. This distinct characteristic of the temperature patterns during the quiescence should be considered when referring to equation (18). In terms of the physics involved, the noted escalation in imbalance and deviation from linearity in the temperature profiles during quiescence might be associated with the chemical reaction, leading to the amplification of the heat generation rate.

![Fig.2. Basic Temperature profile for various values of $F$](image)

The figures 3 through 5 illustrate the influence of porous media and couple stress on the system's stability. It is apparent that the parameter of interest, the critical Darcy-Rayleigh number, denoted by $R_{dc}$, escalates alongside the $D_a$, $\Lambda$ and $\Gamma$, indicating an acceleration of the system. Consequently, the presence of the porous layer and couple stress contributes to the stabilization of the system.

![Fig. 3. Graph of $R_{dc}$ versus $F$ with different values in $D_a$](image)
Conversely, Figure 6 illustrates the impact of $M_3$ on ferro-convection within a porous medium induced by chemical reactions and couple stress. The parameter symbolizes the deviation toward nonlinearity in magnetization. Figure 6 clearly indicates that the destabilizing impact of $M_3$ is relatively insignificant in the occurrence of a chemical reaction. However, it is plausible that the suppression of convection due to chemical reaction can occur in a ferromagnetic fluid whenever a notable increase in the nonlinearity of magnetization.

Figure 7 portrays the combined influence of magnetic stress and chemical reaction. Figure 7 reveals that $R_{dc}$ increases with the simultaneous decrease of both $F$ and $N$. Consequently, the cusp of Bénard-Brinkman convection in a ferrofluid is heightened by the stresses from both magnetization and chemical reaction mechanisms. Notably, the fluctuations induced by the magnetic mechanism gains prominence solely if the $F$ parameter reaches a significant magnitude. In the limiting scenario of $\Gamma, F$ and $N$ approaching zero, the well-known values of $a_c = \pi$ and $R_{dc} = 4\pi^2$ can be obtained as documented in the literature [25].
Fig. 7. Graph of $R_{ac}$ versus $F$ with different values in $N$

Conclusions

The investigation of instability of the Bénard-Brinkman type induced by the play of magnetic and convective forces, influenced by a non-autocatalytic exothermic reaction and couple stress, is conducted through the stability analysis involving infinitesimally small disturbances utilizing normal modes, subsequent to the application of the higher-order Galerkin method. The analysis of the current study leads to the subsequent conclusions:

- The destabilizing effect of the chemical reaction can be largely attributed to the nonlinearity and asymmetry inherent in the thermal profiles of the basic state.
- Magnetic forces and chemical reactions both contribute to a destabilizing dynamic, and they vie with each other in amplifying the magnitude of instability.
- The stability of the system is strengthened by the interaction between couple stress and porous characteristics.
- The escalation of the convection threshold is attributed to the nonlinearity of magnetization, which diminishes in significance as $M_3$ reaches a significant magnitude.

The findings of this study could significantly impact engineering applications related to heat transfer, especially in the context of utilizing microfluidic devices.

References


