Journal of Chemistry and Technologies

pISSN 2663-2934 (Print), ISSN 2663-2942 (Online)

journal homepage: http://chemistry.dnu.dp.ua

UDC 6 81.11.031.12:519.673

MODELING OF THE VISCOPLASTIC FLOW OF A BINGAM FLUID WITH TRANSVERSE CIRCULATION IN A RECTANGULAR CHANNEL OF A WORM MACHINE

Eduard V. Biletsky,[1] , Elena V. Petrenko,[2] Dmitrij P. Semeniuk[2]

[1] *Kharkiv institute of Trade and Economy of Kyiv National University of Trade and Economy, Kharkiv, Ukraine, 8 side-str. O.Yarosha, Kharkiv, 61045, Ukraine*

[2] *Kharkiv State University of Food Technology and Trade, 333 Klochkovskaya, City, Kharkiv, 61051, Ukraine*

Received 20 February 2019; accepted 18 December 2019; available online 15 January 2020

Abstract

Introduction. We proposed a mathematical description of the flow of a Bingham fluid with transverse circulation in a rectangular channel of a worm machine. Materials and methods. As a material, we chose Bingham fluids with two constant parameters - viscosity and fluidity thresholds. We overviewed the influence of the transverse circulation on such characteristics of the visco-plastic flow as the size of the solid core, velocity of the core and the flow rate. The flow in a rectangular channel is formed as a result of superposition of flows in two plane channels which are crossed at right angles. Results. During the simulation of the three-dimensional flow of Bingham fluid in the channel of a rectangular cross-section with transverse circulation, two basic elements are applied. The first one consists of dividing the rectangle into a solid core and four rectangular sections of the viscous flow. The second element is that the viscous flow in each of the plots is two-dimensional, that is, longitudinal and transverse, but depends on the one coordinate. This means that such flows are equivalent to the flows with transverse circulation in the flat channel, the core of the flow of the Bingham fluid has a rectangle at the intersection. This approach allows to calculate all the basic characteristics of a complex three-dimensional flow in the explicit analytical form and analyze its dependence on the boundary conditions, taking into account the influence of all eight longitudinal and transverse conditions with any possible distribution on the boundaries of the channel. We proposed the calculation formulas for determining the velocity of the core of the current, flow rates and the values of dissipation energy in the symmetric form with respect to the coordinates. Conclusion. The mathematical description of the longitudinal flow of a Bingham fluid with transverse circulation in a rectangular channel of a worm machine allows to carry out the simulation of various flows of visco-plastic liquids and to determine the macro-kinetic characteristics at each point of the channel.

Keywords: Bingham fluid; flow; circulation; model; channel; worm machine.

МОДЕЛЮВАННЯ В'ЯЗКОПЛАСТИЧНОЇ ТЕЧІЇ БІНГАМОВСЬКОЇ РІДИНИ З ПОПЕРЕЧНОЮ ЦИРКУЛЯЦІЄЮ В ПРЯМОКУТНОМУ КАНАЛІ ЧЕРВ'ЯЧНОЇ МАШИНИ

Едуард В. Білецький, [1] Олена В. Петренко, [2] Дмитро П. Семенюк[2]

[1] *Харківський торгово-економічний інститут Київського національного торгово-економічного університету, пров. Отакара Яроша, 8, Харків, 61045, Україна*

[2] *Харківський державний університет харчування та торгівлі вул. Клочківська, 333, Харків 61051, Україна*

Анотація

Запропоновано математичний опис течії бінгамовської рідини з поперечною циркуляцією в прямокутному каналі черв'ячної машини. Як матеріал обрано бінгамовські рідини з постійними двома параметрами – в'язкістю та порогом текучості. Розглянуто вплив поперечної циркуляції на такі характеристики в'язкопластичної течії, як розміри твердого ядра, швидкість ядра й витрати рідини течії. При моделюванні тривимірної течії бінгамівської рідини в каналі прямокутного поперечного перерізу з поперечною циркуляцією застосовано два основних елементи. Перший полягає в розбивці прямокутника на тверде ядро та чотири прямокутні ділянки в'язкої течії. Другий елемент полягає в тому, що в'язка течія у кожній із ділянок є двомірною, тобто поздовжньою і поперечною, але залежить від однієї координати. Це означає, що такі течії еквівалентні течіям із поперечною циркуляцією в пласкому каналі, ядро течії бінгамовської рідини має в перетині прямокутник. Даний підхід дозволяє в явному аналітичному виді обчислити всі основні характеристики складної тривимірної течії й проаналізувати її залежність від граничних умов із урахуванням впливу усіх восьми поздовжніх і поперечних умов із будь-яким можливим їхнім розподілом на границях каналу. Запропоновано розрахункові формули для визначення швидкості ядра течії, витрати течії, величини енергії дисипації в симетричному вигляді відносно координат.

Ключові слова: бінгамовська рідина, течія, циркуляція, модель, канал, черв'ячна машина.

**Corresponding author: tel.: +380677542916; e-mail: elenalion71@gmail.com*

doi: 10.15421/081921

Journal of Chemistry and Technologies, 2019, *27*(2), 212-222

МОДЕЛИРОВАНИЕ ВЯЗКОПЛАСТИЧЕСКОГО ТЕЧЕНИЯ БИНГАМОВСКОЙ ЖИДКОСТИ С ПОПЕРЕЧНОЙ ЦИРКУЛЯЦИЕЙ В ПРЯМОУГОЛЬНОМ КАНАЛЕ ЧЕРВЯЧНОЙ МАШИНЫ

Эдуард В. Белецкий,[1] Елена В. Петренко,[2] Дмитрий П. Семенюк[2]
[1]*Харьковский торгово-экономический институт Киевского национального торгово-экономического университета , пер. Отакара Яроша, 8, Харьков, 61045, Украина*
[2]*Харьковский государственный университет питания и торговли, ул. Клочковская, 333, Харьков, 61051*, *Украина*

Аннотация

Предложено математическое описание течения бингамовской жидкости с поперечной циркуляцией в прямоугольном канале червячной машины. В качестве материала выбраны бингамовские жидкости с двумя постоянными параметрами – вязкостью и порогом текучести. Рассмотрено влияние поперечной циркуляции на такие характеристики вязкопластического течения, как размеры твердого ядра, скорость ядра и расход жидкости течения. При моделировании трехмерного течения бингамовской жидкости в канале прямоугольного поперечного сечения с поперечной циркуляцией применены два основных элемента. Первый заключается в разбивке прямоугольника на твердое ядро и четыре прямоугольных участка вязкого течения. Второй элемент состоит в том, что вязкое течение в каждом из участков является двухмерным, то есть продольным и поперечным, но зависит от одной координаты. Это означает, что такие течения эквивалентны течениям с поперечной циркуляцией в плоском канале, ядро течения бингамовской жидкости имеет в сечении прямоугольник. Данный подход позволяет в явном аналитическом виде вычислять все основные характеристики сложного трехмерного течения и проанализировать его зависимость от граничных условий с учетом влияния всех восьми продольных и поперечных условий с любым возможным их распределением на границах канала. Предложены расчетные формулы для определения скорости ядра течения, расхода течения, величины энергии диссипации в симметричном виде относительно координат.

Ключевые слова: бингамовская жидкость; течение; циркуляция; модель; канал; червячная машина.

Introduction

In the food and chemical technology competent instrumentation plays an important role in the processing of raw materials. Machines and apparatus, suitably designed and constructed, are capable of carrying out the corresponding processes as close to the laboratory conditions as possible.

The laws of the flow of high viscosity materials lie at the heart of many technological aspects of processing the raw food materials. The characteristics of the flows of high-viscosity non-Newtonian materials are fundamental to the proper conduction of such processes.

Such movements generate fields of velocity and shear stress, which act as a driving force and means, which effect the materials and ensure the progress of mechanophysical and mechanochemical transformations [1–3]. A movement in which the fields of velocity and stress are generated can not be provided by the pumps due to the high viscosity of the processed materials.

For this purpose the worm machines, specifically designed to influence such materials [4–8] are used in industry. These machines combine the ability to move material with the ability to mix it. These two abilities are prerequisites for effective mechanophic and mechanochemical effects on the material.

Analysis of recent research and publications. As a rule, the transportation ability and the ability of the force (of the shear) impact on the material contradict each other to a certain extent. To optimally combine these abilities, it is necessary to choose the correct structure of the working chamber of the worm machine, which is a collection of serially connected channels with different geometries. In the case of the use of two or more worm machines this statement is maintained, but it should be taking into account that the channels communicate with each other [2; 4; 6].

The channels are distinguished by a large variety of their cross-sections [3–5]. When analyzing the motion in a channel of different shapes, we must locate the calculations on a channel with rectangular cross-section. Such channel acts as the main one, often being the base for various constructions. The flow in such channel is associated with the real movement of the processed material with the help of specially assigned boundary conditions [1; 2; 8]. Namely, on the boundaries of a rectangular channel, the velocities of these boundaries are set, which have both longitudinal (along the length of the channel) and transverse components. The values of these components are determined by the diameter of the worm, the number of revolutions of the worm and the angle of the helix [2; 4; 6]. Boundary velocities determine the velocity and stress fields

Journal of Chemistry and Technologies, 2019, *27*(2), 212-222

within the channel, linking them to the main constructive (worm diameter and pitch) and mode (worm speed) characteristics of the worm machine.Today the study of flow of visco-plastic materials in the channels with different geometric profiles is an actual problem and is of practical interest. However, it should be noted that the study of "simple" one-dimensional models do not fully meet the real flow conditions so that it does not take into account all the functional relationship between the main parameters of the process. Phenomenological models, which describe these conditions in detail, appear to be complex and consider only one longitudinal direction of the flow, excluding boundary velocities. This does not correspond to actual conditions of processes in chemical and food production [3–6; 8]. Taking into account the above-mentioned problems and considering the difference in the pressure at the ends of the channel and the moving boundaries, it should be noted that to date, the simulation of the flow of non-Newtonian fluids in channels with basic geometry is quite an actual and completely unsolved scientific and applied issue.

The purpose of this paper is to study the motion of the Bingam material in a rectangular channel with an arbitrary distribution of the longitudinal and transverse components of the boundary velocities. The choice of such material is dictated by the significant practical importance of such materials and their prevalence in food and chemical technologies [3–5, 8–10].

Tasks of the researches

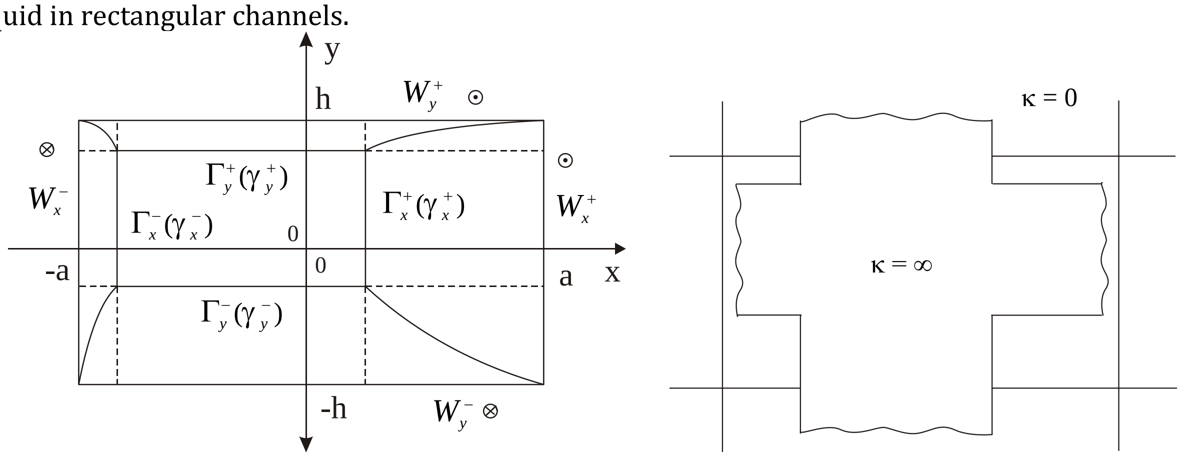
1. The study of macrodynamic and macrokinetic parameters in the flow of Bingam liquid in rectangular channels.
2. Estimation of the influence of transverse circulation on such characteristics of viscoplastic flow as the dimensions of the solid core, velocity of the core and the flow rate for different boundary conditions.
3. Construction of a three-dimensional model of a Bingam fluid in a channel of a rectangular cross-section by various methods, which allows to calculate in an explicit analytical form all the main characteristics of a complex three-dimensional flow and see how it depends on the boundary conditions.

Materials and methods

In this paper, the Bingam material is represented by a Bingham fluid, which has two constant parameters - the viscosity μ and the yield point τ_0. The modeling method is based on the work of the authors, in which they overview the method of flow analysis: the flow in a rectangular channel is reduced to the flow in a plane channel [11–15]. The flow in a rectangular channel is obtained as a result of superposition of flows in the two flat channels crossed at right angles [9; 16–18].

Results of the research and their discussion

The cross-section of a rectangular channel and the characteristics of the core of the flow are shown on the Fig. 1a. The flow in a rectangular channel with moving boundaries is assumed to be three-dimensional and flat. This means that all three velocity components – longitudinal *vz* and transverse *vx* and *vy* depend only on the transverse coordinates – *x* and *y*.



a b

Fig. 1. Model of viscoplastic flow in a rectangular channel

The equations of the motion of a viscoplastic fluid in stresses are as follows:

$$\frac{\partial P}{\partial z} = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y}; \quad \gamma_y^{\pm} \equiv \frac{\Gamma_y^{\pm}}{h}; \; \gamma_x^{\pm} \equiv \frac{\Gamma_x^{\pm}}{a}$$

$$\tau_0^2\left(\gamma^{\pm}_{x(y)}\right) = \tau_{zx}^2 + \tau_{zy}^2 + 2\tau_{xx}^2 + 2\tau_{zy}^2 , \qquad (1)$$

in which P is the pressure in the channel, $\tau_{ik,i,k}$= *x,y,z* are the components of the stress tensor; $\gamma^{\pm}_{x(y)}$ – dimensionless components of the boundary contour of the flow core; a and h are the width and height of the rectangle in the cross section of the channel.

Equations (1) can be successively reduced to flow equations in a flat channel, which has in the cross section a strip parallel to the *OX* axis and in a flat channel, the strip in the section of which is parallel to the *OY* axis. Accordingly, the channels are shown in the Fig. 1b. It is not necessary to consider the flow in two flat channels separately, due to the duality relations between them, it is sufficient to consider only one of the two flows [11; 19]. Without loss of generality, we consider the flow in a flat channel, whose boundaries are parallel to the axis *ax*. The stress tensor τ_{zy} is used as the main component, relative to which the remaining components are estimated. Proceeding from this equation (1) can be transformed:

$$\frac{\partial P}{\partial z} = \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial x}\kappa\rho^{\pm}_{y}(1\mp\gamma^{\pm}_{y}) \, ; \; \frac{\tau_{zx}}{\tau_{zy}} = \frac{\partial \upsilon_z / \partial x}{\partial \upsilon_z / \partial y};$$

$$\tau_0^2 = \tau_{zy}^2\left[1+\left(\frac{\tau_{zx}}{\tau_{zy}}\right)^2 + 2\left(\frac{\tau_{xx}}{\tau_{zy}}\right)^2 + 2\left(\frac{\tau_{xy}}{\tau_{zy}}\right)^2\right];$$

$$\rho^{\pm}_{y} = \frac{W^{+}_{||x} + W^{-}_{||x} - W^{\pm}_{||} - \upsilon_k}{W^{\pm}_{||y} - \upsilon_k}, \qquad (2)$$

where $W^{+}_{||x}$, $W^{\pm}_{||y}$ – are boundary longitudinal velocities (see Fig. 1a); υ_k – is the motion velocity of a solid core; signs "plus" and "minus" indicate the position above and below the core for a quantity with the index "*y*" and to the right and left of the core for quantities with the index "*x*" respectively.

From formulas (2) it folllows that for the subsequent solution it is necessary to exclude all the stress components except for τ_{zy}, and also in the first equation it should be expressed the derivative of τ_{zy} in x in terms of the derivative of τ_{zy} in *y*. The derivatives with respect to these variables are related by the relation $\partial/\partial x = \kappa\, \partial/\partial y$, where $\kappa = h/a$ [11; 17]. Estimates for the components of the stress tensor are as follows:

$$\frac{\tau_{xx}}{\tau_{zy}} = \kappa\frac{W^{+}_{\perp x} - W^{-}_{\perp x}}{W^{\pm}_{||y} - \upsilon_k}(1\mp\gamma^{\pm}_{y});$$

$$\frac{\tau_{xy}}{\tau_{zy}} = \frac{W^{\pm}_{\perp y}}{W^{\pm}_{||y} - \upsilon_k} + \frac{W^{+}_{\perp x} - W^{-}_{\perp x}}{W^{\pm}_{||y} - \upsilon_k}(1\mp\gamma^{\pm}_{y})^2,$$

where $W^{\pm}_{x(y)}$ – are transverse components of the boundary velocities at the boundaries of a rectangular channel parallel to the axes ox and oy respectively. Using the relations (3), the system of equations (1) can be written in the form corresponding to the flow in a plane channel:

$$\frac{\partial P}{\partial z} = \frac{\partial \tau_{zy}}{\partial y}\left[1+\kappa^2\rho^{\pm}_{y}(1\mp\gamma^{\pm}_{y})\right];$$

$$\tau_{zy} = \pm\frac{\tau_0}{\sqrt{1+\kappa^2\left[(\rho^{\pm}_{y})^2+(\theta^{\pm}_{y})^2\right](1\mp\gamma^{\pm}_{y})^2+\left[m^{\pm}_{y}+n^{\pm}_{y}\kappa^2(1\mp\gamma^{\pm}_{y})^2\right]^2}}, \qquad (4)$$

where $\rho^{\pm}_{y}$ is defined by formula (2), equals to the multiplier which is formed by the boundary velocities when estimating the ratio τ_{xx}/τ_{zx} in formula (3), $m^{\pm}_{y}$ equals to the first addendum, $n^{\pm}_{y}$ – equals to the second addendum, which are formed by boundary velocities to assess the relationship τ_{xy}/τ_{zy} in formula (3). Equation system (4) is solved in works of authors [10; 11]. It is established there that the velocity of the solid core υ_k is a fractional-rational function of the values $\gamma^{\pm}_{y}$. It is also established that the profile of the longitudinal velocity υ_z depends quadratically on the transverse coordinate *y*. The expressions for υ_k and υ_z include three constants, after elimination of which a system of two nonlinear equations is obtained to find the values $\gamma^{\pm}_{y}$. The features indicated here are relevant purely to longitudinal flow in a rectangular channel. The results obtained for purely longitudinal flow can be transferred to a flow with transverse circulation, making corresponding changes in the results of the authors [13]. Since the cause of transverse circulation is the presence of transverse components at the boundary velocities, since in the case $W^{\pm}_{x(y)} \equiv 0$ there is no circulation; and the following equalities hold $\theta^{\pm}_{y} = 0$, $m^{\pm}_{y} = 0$, $n^{\pm}_{y} = 0$. Then the system of equations (4) coincides with the one studied previously [9; 10; 14]. It was shown that the solution of the system of equations for the boundaries of the core $\gamma^{\pm}_{y}$ and $\gamma^{\pm}_{x}$ for flow without transverse circulation is the sum of two solutions for plane channels with sides in sections parallel to the axes *OX* and *OY* respectively. Such flat channels correspond to the limiting values of the channel shape parameter $\kappa = 0$ and $\kappa = \infty$. The case $\kappa = 0$ means that *h=const, a=∞*; and the case $\kappa=\infty$ means that *h=∞,* and *a=const*. For the flow with transverse circulation, the system of equations for calculating the values $\gamma^{\pm}_{y}$ is written in the following form:

$$\frac{(1-\gamma_y^+)^2}{1+\kappa^2\rho_y^+(1-\gamma_y^+)}-\frac{(1+\gamma_y^-)^2}{1+\kappa^2\rho_y^-(1+\gamma_y^-)}=\frac{2\mu(W_{||y}^+-W_{||y}^-)}{h\,dP/d\zeta_h},$$

$$\zeta_h=z/h,$$

$$\frac{\gamma_y^+}{1+\kappa^2\rho_y^+(1-\gamma_y^+)}-\frac{\gamma_y^-}{1+\kappa^2\rho_y^-(1+\gamma_y^-)}=$$

$$=\frac{\tau_0}{\sqrt{1+\kappa^2\left[(\rho_y^+)^2+(\theta_y^+)^2\right](1-\gamma_y^+)^2+\left[m_y^++\kappa^2 n_y^+(1-\gamma_y^+)^2\right]^2}}+ \quad (5)$$

$$+\frac{\tau_0}{\sqrt{1+\kappa^2\left[(\rho_y^-)^2+(\theta_y^-)^2\right](1+\gamma_y^-)^2+\left[m_y^-+\kappa^2 n_y^-(1+\gamma_y^-)^2\right]^2}}.$$

If the system of equations (5) is considered for the case κ=0, then this is equivalent to the system of equations for a flat channel. This means that the equations (5) for $\kappa=0$ are a consequence of the following system of equations, which after elimination of the constants c_1, $c_2^{\pm}$ are transformed into equations (5) for $\kappa=0$:

$$\upsilon_z^{\pm}=\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}\xi_y^2+\frac{(c_1\mp\tau_{0y})}{\mu}h\xi+c_2^{\pm};$$

$$\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}(\gamma_y^+)^2+\frac{c_1-\tau_{0y}^+}{\mu}h\gamma_y^++c_2^+=\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}(\gamma_y^-)^2+\frac{c_1-\tau_{0y}^-}{\mu}h\gamma_y^-+c_2^-;$$

$$\gamma_y^+\frac{\partial P}{\partial\zeta_h}+c_1=\tau_{0y}^+;\ \gamma_y^-\frac{\partial P}{\partial\zeta_h}+c_1=\tau_{0y}^-;$$

$$\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}+\frac{c_1-\tau_{0y}^+}{\mu}h+c_2^+=W_{||y}^+;$$

$$\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}-\frac{c_1+\tau_{0y}^-}{\mu}h+c_2^-=W_{||y}^-;$$

$$\tau_{0y}^{\pm}=\frac{\tau_0}{\sqrt{1+(m_y^{\pm})^2}};\ m_y^{\pm}=\frac{W_{\perp y}^{\pm}}{W_{||y}^{\pm}-\upsilon_k}. \quad (6)$$

After carrying out a number of transformations, which are cumbersome and are not presented for simplicity, for the solutions of equations (6), we end up with the following expressions:

$$\gamma_y^{\pm}(\kappa=0)=\pm\frac{\tau_{0y}^++\tau_{0y}^-}{2dP/d\zeta_h}+\frac{\mu(W_{||y}^+-W_{||y}^-)}{2h\,dP/d\zeta_h\left(1-\frac{\tau_{0y}^++\tau_{0y}^-}{2dP/d\zeta_h}\right)}. \quad (7)$$

If there is no transverse circulation, then $\tau_0^+=\tau_0^-$, since $m_y^{\pm}$ turn to zero. Using the dual nature of the values characterizing flows in flat mutually perpendicular channels with a width h and a, based on (7), we can immediately write expressions for quantities $\gamma_x^{\pm}$ for $\kappa=\infty$:

$$\gamma_x^{\pm}(\kappa=\infty)=\pm\frac{\tau_{0x}^++\tau_{0x}^-}{2dP/d\zeta_a}+\frac{\mu(W_{||x}^+-W_{||x}^-)}{2a\,dP/d\zeta_a\left(1-\frac{\tau_{0x}^++\tau_{0x}^-}{2dP/d\zeta_a}\right)},$$

$$\zeta_a=\frac{z}{a}. \quad (8)$$

Formulas (7) and (8) give a solution of the system of equations (5) for values $\gamma_y^{\pm}$ at $\kappa=0$ and a similar system of equations for values $\gamma_x^{\pm}$, that differs from (5) by changing indices "y" to indices "x" and by changing $\kappa\to1/\kappa$; which was not presented before in order to avoid repetition. .

Next, we need to solve the system of equations (5) for another limiting case: $\kappa=\infty$, then, using duality considerations, write the expressions for $\gamma_x^{\pm}(\kappa=0)$. The system of equations (5) in the limit $\kappa=\infty$ takes the following form:

$$\begin{cases}\dfrac{1-\gamma_y^+}{\rho_y^+}-\dfrac{1+\gamma_y^-}{\rho_y^-}=\dfrac{2\mu}{a}\dfrac{W_{||y}^+-W_{||y}^-}{dP/d\zeta_a},\\[2ex] \dfrac{\gamma_y^+}{\rho_y^+(1-\gamma_y^+)}-\dfrac{\gamma_y^-}{\rho_y^-(1+\gamma_y^-)}=0.\end{cases} \quad (9)$$

It can be shown that the system of equations (9) is equivalent to one quadratic equation with respect to $\gamma_y^{\pm}$. Omitting the long and cumbersome transformations, the result of the system of equations (9) can be written in the following form:

$$\frac{1\mp\gamma_y^{\pm}}{\rho_y^{\pm}}=-\frac{1}{2}\left[\pm\tau_{wy}-\frac{(\rho_y^+)^2+(\rho_y^-)^2}{\rho_y^+\rho_y^-(\rho_y^++\rho_y^-)}\right]+\sqrt{\left[\pm\tau_{wy}-\frac{(\rho_y^+)^2+(\rho_y^-)^2}{\rho_y^+\rho_y^-(\rho_y^++\rho_y^-)}\right]^2+\tau_{wy}\frac{\rho_y^+}{\rho_y^-(\rho_y^++\rho_y^-)}}$$

$$\tau_{wy}\equiv\frac{2\mu}{a}\frac{W_{||y}^+-W_{||y}^-}{dP/d\zeta_a}. \quad (10)$$

The specific of the solution (10) is the fact that it does not depend on the transverse flow characteristics, which are concentrated in the values $m_y^{\pm}$ and $n_y^{\pm}$. In other words, the values $\gamma_y^{\pm}(\kappa=0)$ depend on the transverse boundary velocities, and the values $\gamma_y^{\pm}(\kappa=0)$ do not depend on them. The general (not for this limiting case) dependency $\gamma_y^{\pm}$ on $m_y^{\pm}$ and $n_y^{\pm}$ is present. This can be verified if the system of equations (5) is expanded in a series on the parameter $1/\kappa$ near the point $\kappa=\infty$ $(1/\kappa=0)$. In the first and subsequent terms, such a relationship is present.In view of the already mentioned dual property between $\gamma_y^{\pm}(\kappa)$ and $\gamma_x^{\pm}(1/\kappa)$, an expression for the values

$\gamma_x^{\pm}(\kappa=0)$ can immediately be written down. To do this, we need to replace all "*x*" indices with "*y*" indices in formula (10); also in the value τ_{wx} (which replaces τ_{wy}) we need to replace the factor $1/a$ with factor $1/h$; in the derivative of the pressure, we must replace the variable ζ_a with the variable ζ_h. Taking into account what has been said, it is now possible to construct expressions for $\gamma_y^{\pm}(\kappa)$ and $\gamma_x^{\pm}(\kappa)$ in the entire range of variation of this parameter, relying on expressions (7), (8), (10) and its analogue for $\gamma_x^{\pm}(\kappa=0)$. Bearing in mind that equations (5) and their analogue for $\gamma_x^{\pm}$ contain only a dependence on κ^2, it should be concluded that the exact solution for equations (5) and their analogues for $\gamma_x^{\pm}$ depends only on κ^2. Therefore, any interpolating formula based on the limiting values (7), (10) should consist of factors that depend only on κ^2. The simplest of the corresponding interpolations has the following form:

$$\gamma_y^{\pm}(\kappa)=\gamma_y^{\pm}(\kappa=0,W_{\|y}^{\pm},W_{\|x}^{\pm},W_{\perp y}^{\pm})\frac{1}{1+\kappa^2}+\gamma_y^{\pm}(\kappa=\infty,W_{\|y}^{\pm},W_{\|x}^{\pm})\frac{\kappa^2}{1+\kappa^2};$$
$$\gamma_x^{\pm}(\kappa)=\gamma_x^{\pm}(\kappa=0,W_{\|x}^{\pm},W_{\|y}^{\pm})\frac{1}{1+\kappa^2}+\gamma_x^{\pm}(\kappa=\infty,W_{\|x}^{\pm},W_{\|y}^{\pm},W_{\perp x}^{\pm})\frac{\kappa^2}{1+\kappa^2}. \quad (11)$$

To calculate the velocity of a solid core and the flow rate it is sufficient to use formulas (7), (8), (10) and its analogue for $\gamma_x^{\pm}(\kappa=0)$, and (11). To calculate the dissipation energy density, we need to add formulas for transverse (circulation) flows. The velocity of the longitudinal flow in the regions located between the boundaries of the solid core $\gamma_y^{\pm}$ and the boundaries of the rectangular channel $y=\pm h$ are the result of solving the first equation – the flow equation in formulas (2). This equation for the longitudinal velocity υ_z is a second-order equation with respect to the single variable *y*. The pressure gradient is constant in this case. If we use the dimensionless variable $\xi_h=y/h$, then the velocity profile υ_z is a square trinomial. The velocity of the longitudinal flow υ_z for the regions located between the boundaries of the solid core $\gamma_x^{\pm}$ and the boundaries of the rectangle with coordinates $x=\pm a$ is also a square trinomial depending on the variable $\xi_a=x/a$.

The boundaries between regions with the dependence of the longitudinal velocity on the variable ξ_h and ξ_a are the curves passing through the vertices of the rectangle of the core and the rectangle in the cross section of the channel. The equations of these curves are given in the work of the authors [9]. The flow calculation procedure is reduced to calculating the costs in each of the four areas shown in the Fig. 1a.

All these areas of the same type are trapezoids with curvilinear sides. The total flow in the channel is equal to the sum of the expenditures in the areas indicated above and the consumption of the solid core. The latter is equal to the product of the area of the core by its velocity. As shown by the authors, the flow rate of a longitudinal flow without transverse circulation is a fractional-rational expression including the values $\gamma_x^{\pm}$ and $\gamma_y^{\pm}$ in the first to fourth powers. The flow rate of a flow with a transverse circulation has the form coinciding with the type of flow without circulation. All the differences are concentrated in the values $\gamma_x^{\pm}$ and $\gamma_y^{\pm}$, which, in accordance with (11), contain the transverse boundary velocities $W_x^{\pm}$, $W_y^{\pm}$ as independent arguments.

The velocity of the longitudinal flow with transverse circulation can be written in the following form:

$$\upsilon_z^{\pm}=-\frac{h}{2\mu}\frac{\partial P}{\partial\zeta_h}\frac{1-\xi_y^2}{1+\kappa^2\rho_y^{\pm}(1\mp\gamma_y^{\pm})}\pm\frac{h}{\mu}\frac{\partial P}{\partial\zeta_h}\frac{\gamma_y^{\pm}(1\mp\xi_y)}{1+\kappa^2\rho_y^{\pm}(1\mp\gamma_y^{\pm})}+W_{\|y}^{\pm}$$
$$,+:\ \gamma_y^{+}\le\xi_x\le1,-:\ -1\le\xi_y\le\gamma_y^{-},$$
$$\upsilon_z^{\pm}=-\frac{a}{2\mu}\frac{\partial P}{\partial\zeta_a}\frac{1-\xi_x^2}{1+(1/\kappa^2)\rho_x^{\pm}(1\mp\gamma_x^{\pm})}\pm\frac{a}{\mu}\frac{\partial P}{\partial\zeta_a}\frac{\gamma_x^{\pm}(1\mp\gamma_x^{\pm})}{1+\kappa^2\rho_x^{\pm}(1\mp\gamma_x^{\pm})}+W_{\|x}^{\pm}$$
$$,+:\ \gamma_x^{+}\le\xi_x\le1,-:\ -1\le\xi_x\le\gamma_x^{-},$$
$$\xi_y=\frac{y}{h};\ \xi_x=\frac{x}{a};\ \xi_h=\frac{z}{h};\ \xi_a=\frac{z}{a}. \quad (12)$$

The velocities υ_x and υ_y of the transverse flow in a rectangular channel should also be sought in the form of a square expression of the variables ξ_x and ξ_y. The corresponding transverse gradients are the sought values, in contrast to the longitudinal pressure gradient $\partial P/\partial z$, which is an independent constant value. To find the transverse velocities and pressure gradients, we must write down the equations of transverse flow for the same regions as for the longitudinal flow. These equations have the following form:

$$\frac{\partial P}{\partial x}=\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z},\ \tau_0^2=\tau_{zx}^2+\tau_{zy}^2+2\tau_{xx}^2+2\tau_{xy}^2$$
$$\frac{\partial P}{\partial y}=\frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}. \quad (13)$$

For definiteness, first consider transverse flows for regions defined by the inequalities $\gamma_y^+ \le \xi_x \le 1$, $-1 \le \xi_y \le \gamma_y^-$. In this case, the independent variable for the flow velocity $\upsilon_x^\pm$ will be ξ_y, and the pressure will depend on the variable ξ_x. Directly from the Fig. 1a follows that the ratios of the components τ_{xx}/τ_{xy}, τ_{xz}/τ_{xy} are equal to:

$$\frac{\tau_{xx}}{\tau_{xy}} = \frac{\partial \upsilon_x^\pm / \partial x}{\partial \upsilon_x^\pm / \partial y + \partial \upsilon_x^\pm / \partial x} = \frac{\kappa (W_{\perp x}^+ - W_{\perp x}^-)(1 \mp \gamma_y^\pm)}{W_{\perp y}^+ + \kappa^2 (W_{\perp x}^+ - W_{\perp x}^-)(1 \mp \gamma_y^\pm)^2} \equiv \kappa \rho_{yx}^\pm (1 \mp \gamma_y^\pm) ;$$

$$\frac{\tau_{xz}}{\tau_{xy}} = \frac{\partial \upsilon_z^\pm / \partial x}{\partial \upsilon_x^\pm / \partial y + \partial \upsilon_y^\pm / \partial x} = \frac{\kappa (W_{||x}^+ - W_{||x}^-)(1 \mp \gamma_y^\pm)}{W_{\perp y}^+ + \kappa^2 (W_{\perp x}^+ - W_{\perp x}^-)(1 \mp \gamma_y^\pm)^2} \equiv \kappa \rho_{zy}^\pm (1 \mp \gamma_y^\pm) . \qquad (14)$$

If the values of τ_{xx} and τ_{xz} expressed in terms of τ_{xy} in accordance with (14) are substituted into the first of equations (13), then we obtain a partial differential equation connecting $\partial P/\partial x$ and partial derivatives of τ_{xy}. Using the estimates $\partial/\partial x \approx \kappa \partial/\partial y$; $\partial/\partial z \sim \kappa_{zh} \partial/\partial y$, where $\kappa_{zh}=h/L$ for the value of τ_{xy}, the following equation is obtained:

$$\frac{\partial P_y^\pm}{\partial \zeta_a} = \frac{\partial \tau_{xy}}{\partial \xi_h} \left[1 + \kappa^2 \rho_{yx}^\pm (1 \mp \gamma_y^\pm) + \kappa \kappa_{zh} \rho_{zy}^\pm (1 \mp \gamma_y^\pm) \right], \qquad (15)$$

where $P_y^\pm$ – is pressure in regions above the flow core.

For the regions defined by the inequalities $\gamma_x^+ \le \xi_x \le 1$ and $-1 \le \xi_x \le \gamma_x^-$, we must consider the second equation of motion (13). In this case, the independent variable for the velocity $\upsilon_y^\pm$ will be ξ_x, and the pressure will depend on the independent variable ξ_y. From the arguments given here, it follows that the stress components τ_{yy} and τ_{yz} should be expressed in terms of the component τ_{yx}. It is easy to do this as $\tau_{yy} = -\tau_{xx}$ due to the equation of flow conservation and the ratio τ_{yx}/τ_y is calculated the same way as in the second formula (14). We can find the corresponding relationship without calculation using the duality relationship. To this end, we need to replace the index "*y*" with the index "*x*" in the formulas (14) and vice versa, and the value κ with $1/\kappa^2$; and introduce the notation $(1/\kappa)\rho_{xy}^\pm (1 \mp \gamma_x^\pm)$, for the ratio τ_{yy}/τ_{yx}, and the notation $(1/\kappa)\rho_{zx}^\pm (1 \mp \gamma_x^\pm)$ for the ratio τ_{yz}/τ_{yx} respectively.

Returning to equation (15), it should be solved with boundary conditions different from the boundary conditions for the longitudinal velocity υ_z. The transverse velocities at the boundaries of the rectangles in the section of the channel must be equal to the transverse velocities of the boundaries. On the surface of a solid core, the transverse velocities should be turned to zero. From formula (15) for pressures $P_y^\pm$ and pressures $P_x^\pm$ for regions to the left and right of the core it follows that $P_y^\pm = P_y^\pm(\zeta_a)$, $P_x^\pm = P_x^\pm(\xi_h)$. The variables ζ_a and ξ_h vary in the following limits: $\gamma_x^+ \le \xi_h \le 1$; $-1 \le \xi_h \le \gamma_x^-$; $\gamma_x^+ \le \zeta_a \le 1$; $-1 \le \zeta_a \le \gamma_x^-$. Consequently, the velocities of the transverse flow must depend on the values $dP_y^\pm / d\zeta_a$ и $dP_x^\pm / d\xi_h$ the same way as on the parameters we are seeking. To find unknown transverse pressure gradients, we should use these auxiliary representations. The flow between the solid core and the channel boundaries in the transverse plane can be imagined as a flow in four successively connected channels of different widths and lengths with moving and fixed boundaries. In each of these channels, a total flow consisting of a flowing current and a pressure flow is realized. The flowing current is caused by the movement of the corresponding boundary, and the pressure flow is caused by the transverse pressure drops. The four mentioned channels form a closed channel. This means that the pressure difference along the contour of the four channels must turn to zero. Since the transverse channels are sequentially connected in a closed loop, the costs of the transverse currents in each channel are equal to each other. The condition for the uniformity of all expenditures gives three conditions for the unknown values of the pressure gradients. Another condition appears due to the fact that the pressure drop along the closed circuit of the channels is zero. An additional unknown parameter is the pressure at the starting point of the closed contour (it is also its final point). The choice of the starting point is arbitrary. It does not affect the results. Below, the point with the coordinates $\zeta_a=1$ and $\xi_h=1$ is used as the initial point for pressure. To determine the pressure at this point, the following condition should be used: the average value of the pressure in the cross-section of the rectangular channel must be equal to the value of the longitudinal pressure in this section [9; 17]. The longitudinal pressure is a linear function of the longitudinal coordinate *z* and the boundary conditions at the ends of the channel. The longitudinal pressure is set independently by indicating the pressure values

Journal of Chemistry and Technologies, 2019, *27*(2), 212-222

P_h and P_k. It follows that the transverse pressure is a function of the longitudinal coordinate z, the values $P_h(z=0)$ and $P_k(z=L)$, and the values of the transverse velocities of the boundaries $W_y^{\pm}$, $W_x^{\pm}$. Another group of parameters that determine the behavior of the transverse pressure is associated with the characteristics of the longitudinal flow. These characteristics are: $\gamma_y^{\pm}$, $\gamma_x^{\pm}$, $\rho_{yx}^{\pm}$, $\rho_{zy}^{\pm}$, which, in turn, depend on μ, τ_0, $W_{\|y}^{\pm}$, $W_{\|x}^{\pm}$. Thus, the transverse pressure depends on all the boundary velocities and the coordinates of the boundaries of the solid core and the shape parameter κ. After these remarks, we can start deriving the equations for the lateral pressure and the flow. All the above is shown on the Fig. 2.

The solution of equations (15) for $\upsilon_x^{\pm}(\xi_h)$ and its analogue for $\upsilon_y^{\pm}(\xi_a)$ in the above-mentioned limits with the boundary conditions discussed above is obtained after a series of cumbersome transformations and double integration over the transverse variable ξ_h for $\upsilon_x^{\pm}$ and ζ_a for $\upsilon_y^{\pm}$.

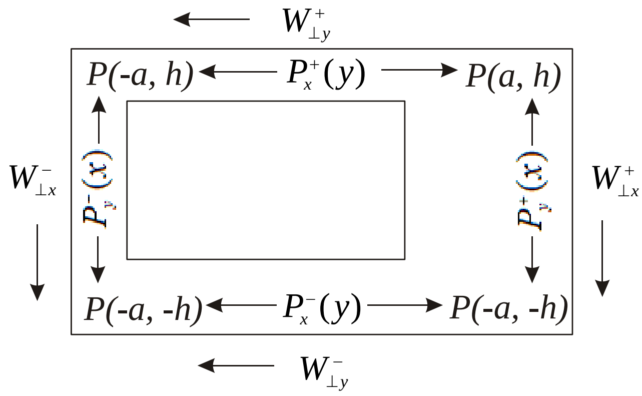


Fig. 2. Characteristics of the flow core

The result is as follows:

$$\upsilon_x^{\pm}(\xi_h)=\frac{h}{2\mu}\frac{1}{1+\kappa^2R^{\pm}(1\mp\gamma_y^{\pm})}\frac{\partial P_y^{\pm}}{\partial\zeta_a}\left[\xi_h^2-\gamma_y^{\pm 2}\pm(1\pm\gamma_y^{\pm})(\gamma_y^{\pm}-\xi_h)\right]\pm\frac{W_{\perp y}^{\pm}(\xi_h-\gamma_y^{\pm})}{(1\mp\gamma_y^{\pm})};$$

$$\upsilon_y^{\pm}(\zeta_a)=\frac{a}{2\mu}\frac{1}{1+\frac{1}{\kappa^2}S^{\pm}(1\mp\gamma_x^{\pm})}\frac{\partial P_x^{\pm}}{\partial\zeta_h}\left[\zeta_a^2-\gamma_x^{\pm 2}\pm(1\pm\gamma_x^{\pm})(\gamma_x^{\pm}-\zeta_a)\right]\pm\frac{W_{\perp x}^{\pm}(\xi_x-\gamma_x^{\pm})}{(1\mp\gamma_x^{\pm})};$$

$$R^{\pm}=\rho_{yx}^{\pm}+\frac{\kappa_{zh}}{\kappa}\rho_{zy}^{\pm};\; S^{\pm}=\rho_{xy}^{\pm}+\frac{\kappa}{\kappa_{za}}\rho_{xz}^{\pm};$$

$$\kappa_{zh}=\frac{h}{L};\; \kappa_{za}=\frac{a}{L}. \qquad (16)$$

Calculation of the flow rate in each channel is performed by integrating the velocity $\upsilon_x^{\pm}(\xi_h)$ in the intervals (γ_y^{+}, 1) and (-1, γ_y^{-});and velocity $\upsilon_x^{\pm}(\zeta_a)$ in the intervals (γ_x^{+}, 1) and (-1, γ_x^{-}). The results of integration are the following:

$$\dot{V}_y^{+}=h\int_{\gamma_y^{+}}^{1}\upsilon_x^{+}d\xi_h=\frac{h^2}{2\mu}\frac{1}{1+\kappa^2R^{+}(1-\gamma_y^{+})}\frac{\partial P_y^{+}}{\partial\zeta_a}\left[\frac{1-3\gamma_y^{+2}+2\gamma_y^{+3}}{3}-\frac{(1+\gamma_y^{+})(1-\gamma_y^{+})^2}{2}\right]+\frac{W_{\perp y}^{+}h}{2}(1+\gamma_y^{+});$$

$$\dot{V}_y^{-}=h\int_{-1}^{\gamma_y^{-}}\upsilon_x^{-}d\xi_h=\frac{h^2}{2\mu}\frac{1}{1+\kappa^2R^{-}(1+\gamma_y^{-})}\frac{\partial P_y^{-}}{\partial\zeta_a}\left[\frac{1-3\gamma_y^{-2}-2\gamma_y^{-3}}{3}-\frac{(1-\gamma_y^{-})(1+\gamma_y^{-})^2}{2}\right]+\frac{W_{\perp y}^{-}h}{2}(1+\gamma_y^{-});$$

$$\dot{V}_x^{+}=a\int_{\gamma_x^{+}}^{1}\upsilon_y^{+}d\zeta_a=\frac{a^2}{2\mu}\frac{1}{1+\frac{1}{\kappa^2}S^{+}(1-\gamma_x^{+})}\frac{\partial P^{+}}{\partial\xi_h}\left[\frac{1-3\gamma_x^{+2}+2\gamma_x^{+3}}{3}-\frac{(1+\gamma_x^{+})(1-\gamma_x^{+})^2}{2}\right]+\frac{W_{\perp x}^{+}h}{2}(1+\gamma_x^{+});$$

$$\dot{V}_x^{-}=a\int_{-1}^{\gamma_x^{-}}\upsilon_y^{-}d\zeta_a=\frac{a^2}{2\mu}\frac{1}{1+\frac{1}{\kappa^2}S^{-}(1+\gamma_x^{-})}\frac{\partial P^{-}}{\partial\xi_h}\left[\frac{1-3\gamma_x^{-2}-2\gamma_x^{-3}}{3}-\frac{(1-\gamma_x^{-})(1+\gamma_x^{-})^2}{2}\right]+\frac{W_{\perp x}^{-}a}{2}(1+\gamma_x^{-}). \quad (17)$$

The fact that the channels of transverse circulation form a closed contour, and the transverse pressure itself must be continuous at the junctions of the channels, leads to the following relations between the pressure values: $P_y^{+}(-a)=P_x^{-}(+h)$, $P_x^{-}(-h)=P_y^{-}(-a)$, $P_y^{-}(+a)=P_x^{+}(-h)$, $P_x^{+}(+h)=P_y^{+}(+a)$. From equation (15) and its analogue for $\upsilon_y^{\pm}(\zeta_a)$ it follows that the dependence of the pressure on the corresponding coordinates along the transverse channels has the following form:

$$P_y^+ = P_0 + \frac{\partial P_y^+}{\partial \zeta_a}(\zeta_a - 1);$$

$$P_x^- = P_0 - 2\frac{\partial P_y^+}{\partial \zeta_a} + \frac{\partial P_x^-}{\partial \xi_h}(\xi_h - 1);$$

$$P_y^- = P_0 - 2\frac{\partial P_y^+}{\partial \zeta_a} - 2\frac{\partial P_x^-}{\partial \xi_h} + \frac{\partial P_y^-}{\partial \zeta_a}(\zeta_a + 1);$$

$$P_x^+ = P_0 - 2\frac{\partial P_y^+}{\partial \zeta_a} - 2\frac{\partial P_x^-}{\partial \xi_h} + 2\frac{\partial P_y^-}{\partial \zeta_a} + \frac{\partial P_x^+}{\partial \xi_h}(\xi_h + 1), \quad (18)$$

where P_0 – is the pressure at the initial point with coordinates *(x = a, y = h).*

The average value of the transverse pressure along a closed contour enclosing a solid core is defined as the sum of the integrals over the variables ξ_h and ζ_a in the appropriate limits. When integrating, the terms with ξ_h and ζ_a give a zero contribution. Therefore, the average value of the transverse pressure is:

$$2P_0 - \frac{3+4\kappa}{1+\kappa}\frac{\partial P_y^+}{\partial \zeta_a} + \frac{1+2\kappa}{1+\kappa}\frac{\partial P_y^-}{\partial \zeta_a} - \frac{1+3\kappa}{1+\kappa}\frac{\partial P_x^-}{\partial \xi_h} + \frac{\kappa}{1+\kappa}\frac{\partial P_x^+}{\partial \xi_h} = P_{\|}(z),$$

$$P_{\|}(z) = \frac{P_K - P_H}{L} Z + P_H. \quad (19)$$

The condition that the pressure difference along the closed contour equal to zero imposes the following restriction on the gradients of the transverse pressures:

$$-\frac{\partial P_y^+}{\partial \zeta_a} + \frac{\partial P_y^-}{\partial \zeta_a} - \frac{\partial P_x^-}{\partial \xi_h} + \frac{\partial P_x^+}{\partial \xi_h} = 0. \quad (20)$$

The set of equations (19), (20) and also the three equations obtained by equating the expressions for the expenses given by formulas (17) compose a system of five linear equations for determining five unknown variables $\partial P_x^{\pm}/\partial \zeta_a$, $\partial P_x^{\pm}/\partial \xi_h$ and P_0. Solving these equations and substituting the results to formulas (16) we can obtain closed expressions for the rates of transverse circulation. After obtaining the velocities $\upsilon_x^{\pm}(\xi_h)$ and $\upsilon_y^{\pm}(\zeta_a)$ it is possible to calculate the energy of dissipation of viscoplastic flow in a rectangular channel taking into account the transverse circulation. The calculation procedure is based on the fact that in each of the four regions to which the rectangle in the channel section is divided, there is a flow that has longitudinal and transverse components. For regions above and below the core these are $\upsilon_z^{\pm}(\xi_h)$ and $\upsilon_x^{\pm}(\xi_h)$; for regions to the left and to the right of the core these are $\upsilon_z^{\pm}(\zeta_a)$ and $\upsilon_y^{\pm}(\zeta_a)$. It should also be taken into account that because of equation (19), it seems that an additional dependence of the values $\upsilon_x^{\pm}$ and $\upsilon_y^{\pm}$ on the longitudinal coordinate z also arises. Therefore, the amount of dissipation energy $\dot{e}$ per cross-section of a rectangular channel must include terms that contain from $(\partial \upsilon_x^{\pm}/\partial z)^2$ and $(\partial \upsilon_y^{\pm}/\partial z)^2$. In fact, there is no such dependence. Equations for consumption include only pressure gradients over transverse coordinates such that the conditions of constant flow and zero pressure variation along the closed contour make up four independent equations for the four gradients. Therefore, although the pressure in the channel depends on all the coordinates *x*, *y*, *z*, the kinematic values depend only on the *x* and *y* coordinates. Considering these considerations, the value can be represented as the following sum:

$$\frac{\dot{e}}{2\mu} = \int_{\Gamma_y^+}^{h}\left(\frac{\partial \upsilon_z^+}{\partial y}\right)^2 dy\cdot 2a + \int_{-h}^{\Gamma_y^-}\left(\frac{\partial \upsilon_z^-}{\partial y}\right)^2 dy\cdot 2a + \int_{\Gamma_x^+}^{a}\left(\frac{\partial \upsilon_z^+}{\partial x}\right)^2 dx\cdot 2h + \int_{-a}^{\Gamma_x^-}\left(\frac{\partial \upsilon_z^-}{\partial y}\right)^2 dx\cdot 2h +$$
$$+ \int_{\Gamma_y^+}^{h}\left(\frac{\partial \upsilon_x^+}{\partial y}\right)^2 dy\cdot 2a + \int_{-h}^{\Gamma_y^-}\left(\frac{\partial \upsilon_x^-}{\partial y}\right)^2 dy\cdot 2a + \int_{\Gamma_x^+}^{a}\left(\frac{\partial \upsilon_y^+}{\partial x}\right)^2 dx\cdot 2h + \int_{-a}^{\Gamma_x^-}\left(\frac{\partial \upsilon_y^-}{\partial y}\right)^2 dx\cdot 2h. \quad (21)$$

The value of the longitudinal flow rate is calculated for each of the regions separately and is also summed up. As a result, the following expression is obtained for consumption $\dot{V}_z$:

$$\dot{V}_z = \int_{\Gamma_y^+}^{h} \upsilon_{zy}^+ dy\cdot 2a + \int_{-h}^{\Gamma_y^-} \upsilon_{zy}^- dy\cdot 2a + \int_{\Gamma_x^+}^{a} \upsilon_{zx}^+ dx\cdot 2h + \int_{-a}^{\Gamma_x^-} \upsilon_{zx}^- dx\cdot 2h, \quad (22)$$

where υ_{zy}^{+} are defined by formulas (8), and $\upsilon_{zx}^{\pm}$ – are defined by their analogues based on duality principle.

Conclusion

Summarizing the results, we can conclude that the model of a three-dimensional flow of a Bingham fluid in a channel with a rectangular cross-section has been constructed with the use of two basic techniques. One technique is to divide the rectangle into a solid core and the four rectangular areas. The second method is that the viscous flow in each of the regions is two-dimensional – longitudinal and transverse, but both currents depend on the one coordinate. This means that such flows are equivalent to flows with transverse circulation in the flat channel. An additional technique arising from the first two ones is that the core of the Bingham fluid flow has a rectangle in its section. This, of course, should be considered as the some approximation to the real situation. This approach is justified by the possibility of the calculating all the main characteristics of a complex three-dimensional flow in an explicit analytical form and it shows how they depend on the boundary conditions. It also takes into account all the eight longitudinal and transverse boundary conditions with any possible distribution on the channel boundaries.

Бібліографічні посилання

[1] M. Anandha Rao. Rheology of Fluid, Semisolid, and Solid Foods: Principles and Applications / Rao M. Anandha. – New York: Imprint: Springer, 2014. – 461 p.
[2] Gale M. Mixing in Single Screw Extrusion / M. Gale. – Smithers Rapra Press, 2009. – 300 p.
[3] Остриков А. Н. Экструзия в пищевой технологии / А. Н. Остриков, О. В. Абрамов, А. С. Рудометкин. – СПб.: Гиорд, 2004. – 288 с.
[4] Ясногородский А. Л. Многоцелевые двухшнековые машины для перерабатывающих технологий / А. Л. Ясногородский, А.Г. Звездин. – Харьков: Прапор, 2006. – 193 с.
[5] Раувендаль К. Экструзия полимеров / К. Раувендаль. - СПб.: Профессия, 2006. – 768 с.
[6] Колгрюбер К. Двухшнековые сонаправленные экструдеры. Основы, технология, применение / К. Колгрюбер – СПб.:.: Профессия, 2016. – 352 с.
[7] Single screw extruder range Helibar U [Electronic resource] / The official website company Komax, 2015. – Available at: http:// www.komax.pro/en/extrusion/extruder-long-helical-grooves
[8] Бонтен К. Ресурсоэффективность в технологии пластмасс [Электронный ресурс] / К. Бонтен, Оливер Каст // Institut für Kunststofftechnik, Штутгарт, Германия / По материалам Motiva Group, Хельсинки, 2015. – Режим доступа: http://www.motiva.fi/Файлы / 9729 / 02 Resource efficiency in plastics industry Bonten K ast.pdf
[9] Товажнянський Л. Л. Моделювання течій неньютонівських рідин у каналах базової геометрії / Л. Л. Товажнянський, Е.В. Білецький, Ю. А. Толчинський – Харьков: НТУ «ХПІ», 2013. – 319 с.
[10] Кирсанов Е. А. Неньютоновское течение дисперсных, полимерных и жидкокристаллических систем: структурный подход / Е. А. Кирсанов, В. Н. Матвеенко – Москва: Техносфера, 2016. – 383с.
[11] Ульев Л. М. Ламинарное течение в соосных конических каналах / Л. М. Ульев. – Харьков: НТУ «ХПИ», 2006 – 660 с.
[12] Пономарева М. А. Циркуляционное течение высоковязкой неньютоновской жидкости в канале одношнекового экструдера / М. А Пономарева, М. П. Филина, В. А. Якутенок // Вестник Томского государственного университета. Математика и механика. 2016. – № 2 (40). – С. 97–107.
[13] Перминов А. В. Устойчивость стационарного плоскопараллельного течения псевдопластической жидкости в плоском вертикальном слое / А. В. Перминов, Т. П. Любимова // Вычислительная механика сплошных сред. – 2014. – Т. 7, № 3. – С. 270 – 278.
[14] Cantelli A. Uniform Flow of Modified Bingham Fluids in Narrow Cross Sections / A. Cantelli // Journal of Hydraulic Engineering, 2009. – 135(8). – P. 640–50.
[15] Голованчиков А. Б. Моделирование течения вязкопластической реакционной массы в шнековом реакторе с маловязким пристенным слоем / А. Б. Голованчиков, А. А. Шагарова // Известия вузов. Химия и химическая технология. – 2015. – Т. 58, № 12. – С. 69–72.
[16] Biletsky E. Theoretical aspects of non-newtonian fluids flow simulation in food technologies / E. Biletsky, E. Petrenko, D. Semeniuk // Ukrainian Food Journal. - 2014. – 3(2). – P. 271–280.
[17] Biletsky E. Three-dimensional model of non-Newtonian fluid flow in the rectangular channel / E. Biletsky, E. Petrenko, D. Semeniuk // Ukrainian Food Journal. – 2016. – 5(3). – P. 550–560.
[18] Mitsoulis E. Numerical simulations of complex yield-stress fluid flows / E. Mitsoulis, J. Tsamopoulos // Rheologica Acta. – 2017. – 56 – P. 231–258.
[19] Saramito P. Progress in numerical simulation of yield stress fluid flows / P. Saramito, A. Wachs // Rheologica Acta 56. – 2017. – P. 211–230.
[20] Frigaard I. A. Binghams model in the oil and gas industry / I. A. Frigaard, K. Paso, P. de Souza Mendes // Rheologica Acta. – 2017. – 56. – P. 259–282.

References

[1] Anandha, Rao M. (2014). *Rheology of Fluid, Semisolid, and Solid Foods: Principles and Applications*. New York, USA : Imprint: Springer.
[2] Gale, M. (2009). *Mixing in Single Screw Extrusion.* USA: Smithers Rapra Press.
[3] Ostrikov, A.N., Abramov, O.V., Rudometkin A.S. (2004). [*Extrusion in food technology*]. Saint Petersburg. Russian Federation: Giord. (in Russian).
[4] Yasnogorodsky, A. L. Zvezdin, A.G. (2006). [*Multi-purpose twin-screw machines for processing technologies*]. Kharkov. Ukraine: Prapor. (in Russian).
[5] Rauvendal, K. (2006). [*Extrusion of Polymers*] Saint Petersburg. Russian Federation: Profession. (in Russian).
[6] Kolgruber, K. (2016). [*Twin-screw, co-directed extruders. Basics, technology, application*] Saint Petersburg. Russian Federation: Profession. (in Russian).
[7] The official website company Komax. (2015). [*Single screw extruder range Helibar U*]. http:// www.komax.pro/en/extrusion/extruder-long-helical-grooves

[8] Bonten, K., Oliver Cast (2016). [*Resource efficiency in plastics technology*] Institut für Kunststofftechnik Stuttgart, Germany, According to the materials of the Motiva Group, Helsinki. http: // www. motiva.fi/Files / 9729 / 02 Resource efficiency in plastics industry Bonten K ast.pdf

[9] Tovazhnyansky, L. L., Biletsky, E.V., Tolchinsky Yu. A. (2013) [*Modeling of flows of non-Newtonian fluids in channels of basic geometry*]. Kharkiv. Ukraine: NTU "KhPI". (in Ukraine).

[10] Kirsanov, E. A., Matveenko V. N. (2016). [*Non-Newtonian Current of Disperse, Polymer, and Liquid Crystal Systems*] Moscow. Russian Federation: Technosphere. (in Russian).

[11] Ulyev, L. M. (2016). [*Laminar flow in coaxial conical channels*]. Kharkiv. Ukraine: NTU "KhPI". (in Ukraine).

[12] Ponomareva, M. A. Filina, M. P., Yakutonok, V. A. (2016). [Circulating flow of high viscosity non-Newtonian fluid in a single-screw extruder channel]. *Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika*, *2* (40), 97–107.

[13] Perminov, A. V., . Lyubimova, T. P. (2014). [Stability of a stationary plane-parallel flow of a pseudoplastic fluid in a flat vertical layer]. *Vychislitel'naya mekhanika sploshnykh sred*. *7*(3). 270 – 278. (in Russian).

[14] Cantelli, A. (2009). Uniform Flow of Modified Bingham Fluids in Narrow Cross Sections *Journal of Hydraulic Engineering*. 135(8). 640-50.

[15] Golovanchikov, A. B., Shagarova, A. A. (2015). [*Modeling the flow of a viscoplastic reaction mass in a screw reactor with a low viscosity wall layer*]. Izvestiya vuzov. Khimiya i khimicheskaya tekhnologiya. *58*(12), 69–72. (in Russian).

[16] Biletsky, E. Petrenko, E., Semeniuk, D. (2014). Theoretical aspects of non-newtonian fluids flow simulation in food technologies. *Ukrainian Food Journal*. *3*(2), 271–280.

[17] Biletsky, E. Petrenko, E., Semeniuk, D. (2016). Three-dimensional model of non-Newtonian fluid flow in the rectangular channel. *Ukrainian Food Journal*. *5*(3), 550-560.

[18] Mitsoulis, E., Tsamopoulos, J. (2017). Numerical simulations of complex yield-stress fluid flows. *Rheologica Acta 56*. 231–258.

[19] Saramito, P., Wachs A. (2017). Progress in numerical simulation of yield stress fluid flows. *Rheologica Acta 56*. 211–230.

[20] Frigaard, I. A., Paso, K., de Souza Mendes, P., (2017) Binghams model in the oil and gas industry. *Rheologica Acta 56*. 259–282.