Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

Journal of Chemistry and Technologies

pISSN 2663-2934 (Print), ISSN 2663-2942 (Online)

journal homepage: http://chemistry.dnu.dp.ua

UDC 6 81.11.031.12:519.673

DETERMINATION OF HEAT TRANSFER COEFFICIENTS DURING THE FLOW OF NON-NEWTONIAN FLUIDS IN PIPES AND CHANNELS OF CHEMICAL PROCESS EQUIPMENT

Eduard V. Biletsky,[1] Elena V. Petrenko*,[2] Dmitrij P. Semeniuk[2]

[1] National Technical University «Kharkiv Polytechnic Institute», 2, Kyrpychova str., Kharkiv, 61002, Ukraine
[2] Kharkiv State University of Food Technology and Trade, 333 Klochkovskaya str., Kharkiv, 61051, Ukraine

Received 20 March 2020; accepted 1 June 2020; available online 22 June 2020

Abstract.

The problem of heat transfer of non-Newtonian fluids in the channels of chemical-technological equipment is considered. A mathematical model is proposed for determining heat transfer characteristics during the flow of Bingham fluids, generalized displaced fluids and power fluids in channels of different geometries. During the Bingham fluid flow, for the calculation of heat transfer coefficients, the convective temperature transfer equation is given in the approximation of the thermal boundary layer so that only the transverse derivative with respect to y is stored on the right side, and the x coordinate is assumed to be aligned along the tangent component of the fluid flow velocity. Nusselt numbers are determined by the derivatives of the tangent velocity on the walls of the channels and at the boundaries of the solid core. If the tangent of the fluid velocity on the wall has two components, then the velocity, the derivative of the Nusselt number, is determined through these components in accordance with the Pythagorean theorem. When a generalized shear fluid is used to calculate the Nusselt numbers, it must be taken into account that in a flat channel with longitudinal and longitudinal-transverse flows there are two heat transfer coefficients, and in a rectangular channel there are four heat transfer coefficients. The determination of the heat transfer coefficients of a power-law fluid is considered only for longitudinal flow in a flat channel and is carried out similarly to the calculation procedure for Bingham and generalized-shear fluids. The obtained expressions, when carrying out engineering calculations, allow us to calculate the corresponding heat transfer and heat transfer coefficients during the flow of non-Newtonian fluids in the channels and with the environment.

Keywords: non-Newtonian fluid; flow; heat transfer; pipe; channel; Nusselt number.

ВИЗНАЧЕННЯ КОЕФІЦІЄНТІВ ТЕПЛОВІДДАЧІ ПРИ ТЕЧІЇ НЕНЬЮТОНІВСЬКИХ РІДИН У ТРУБАХ І КАНАЛАХ ХІМІКО-ТЕХНОЛОГІЧНОГО ОБЛАДНАННЯ

Едуард В. Білецький, [1] Олена В. Петренко, [2] Дмитро П. Семенюк[3]

[1] Національний технічний університет «Харківський політехнічний інститут», вул. Кирпичова, 2, Харків, 61002, Україна
[2] Харківський державний університет харчування та торгівлі вул. Клочківська, 333, Харків 61051, Україна

Анотація

Розглянуто проблему теплообміну неньютоновскіх рідин в каналах хіміко-технологічного обладнання. Запропоновано математичну модель визначення теплообмінних характеристик при течії бінгамовських, узагальнено-зрушених та степеневих рідин в каналах різної геометрії. При течії бінгамовської рідини для обчислення коефіцієнтів тепловіддачі рівняння конвективного переносу температури приведено в наближенні теплового прикордонного шару так, що в правій частині збережена тільки поперечна похідна по змінній *y*, а координата *x* вважається спрямованою уздовж дотичної компоненти швидкості течії рідини. Числа Нуссельта визначаються похідними дотичній швидкості на стінках каналів і на кордонах твердого ядра. Якщо дотична швидкості течії рідини на стінці має дві складові, то швидкість, похідна числа Нуссельта, визначається через ці складові відповідно до теореми Піфагора. При течії узагальнено-зрушеної рідини для обчислення чисел Нуссельта необхідно враховувати що в плоскому каналі при поздовжній і поздовжньо-поперечноій течіях є два коефіцієнта тепловіддачі, а в прямокутному каналі – чотири коефіцієнта тепловіддачі. Визначення коефіцієнтів тепловіддачі степеневої рідини розглянуто тільки для поздовжньої течії в плоскому каналі і проводиться аналогічно методики обчислень для бінгамовскої та узагальнено-зрушеної рідин. Отримані вирази, при проведенні інженерних розрахунків дозволяють визнвчати відповідні коефіцієнти тепловіддачі і теплопередачі при течії неньютоновскіх рідин в каналах і з зовнішнім середовищем.

Ключові слова: неньютонівська рідина; течія; теплообмін; труба; канал; число Нуссельта.

**Corresponding author: +380677542916; e-mail: elenalion71@gmail.com*

doi: 10.15421/082010

Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТОВ ТЕПЛООТДАЧИ ПРИ ТЕЧЕНИИ НЕНЬЮТОНОВСКИХ ЖИДКОСТЕЙ В ТРУБАХ И КАНАЛАХ ХИМИКО-ТЕХНОЛОГИЧЕСКОГО ОБОРУДОВАНИЯ

Эдуард В. Белецкий,[1] Елена В. Петренко,[2] Дмитрий П. Семенюк[2]
[1] *Национальный технический университет «Харьковский политехнический институт», ул. Кирпичева, 2, Харьков, 61002, Украина*
[2] *Харьковский государственный университет питания и торговли, ул. Клочковская, 333, Харьков, 61051*, *Украина*

Аннотация
Рассмотрена проблема теплообмена неньютоновских жидкостей в каналах химико-технологического оборудования. Предложена математическая модель определения теплообменных характеристик при течении бингамовских, обобщенно-сдвиговых и степенных жидкостей в каналах разной геометрии. При течении бингамовской жидкости для вычисления коэффициентов теплоотдачи уравнение конвективного переноса температуры приведено в приближении теплового пограничного слоя так, что в правой части сохранена только поперечная производная по переменной *y*, а координата *x* считается направленной вдоль касательной компоненты скорости течения жидкости. Числа Нуссельта определяются производными касательной скорости на стенках каналов и на границах твердого ядра. Если касательная скорости течения жидкости на стенке имеет две составляющие, то скорость, производная числа Нуссельта, определяется через эти составляющие в соответствии с теоремой Пифагора. При течении обобщенно-сдвиговой жидкости для вычисления, чисел Нуссельта, необходимо учитывать, что в плоском канале при продольном и продольно-поперечном течениях есть два коэффициента теплоотдачи, а в прямоугольном канале – четыре коэффициента теплоотдачи. Определение коэффициентов теплоотдачи степенной жидкости рассмотрено только для продольного течения в плоском канале и проводится аналогично методики вычислений для бингамовской и обобщенно-сдвиговой жидкостей. Полученные выражения, при проведенни инженерных расчетов позволяют вычислять соответствующие коэффициенты теплоотдачи и теплопередачи при течении неньютоновских жидкостей в каналах и с окружающей средой.
Ключевые слова: неньютоновская жидкость; течение; теплообмен; труба; канал; число Нуссельта.

Introduction

Heat exchange plays an important role in the processes of the chemical and food industries. A detailed study of the structure of heat flow allows a high level of organization of technological processes. It is known that most liquids used in the production of chemical and food products have an abnormal flow character, so studying the process of heat exchange of non-Newtonian fluids is very relevant. Nowadays, there is little scientific work on the study of heat transfer in non-Newtonian fluids, usually theoretical analytical studies [1]. The nonlinearity of the flow of non-Newtonian fluids creates additional difficulties in solving the problems of convective heat transfer, so very often researchers solve problems in a simplified form and, as a rule, for the laminar mode of motion [2]. It should be noted that turbulent flow conditions are more favorable for the intensification of the heat transfer process, but given that most non-Newtonian fluids are high molecular weight fluids, creating flow turbulence under real conditions is quite a difficult task. When forming a mathematical task for the study of convective heat transfer, a technique is used that leads to the compilation of a complex system of equations. It includes equations of rheological state of material, equation of continuity, energy and equation of thermodynamic composition of liquid [3]. The solution to this problem is the functions that satisfy the specified equation and defined boundary conditions. The boundary conditions include initial conditions consisting of the distribution of velocity, temperature, etc. in the initial time period. If fluid flow and heat transfer are stationary, then the initial conditions are no longer present. The boundary conditions include the geometric shape of the system and the directions of movement and heat transfer. The fluid flow in the pipe is limited by the inner surface of the walls, the inlet and outlet sections, which also constitute boundary conditions. As a rule, the boundary conditions for velocity on the wall surface are set without taking into account the motion of the fluid. The boundary conditions for temperature are formed on the basis of the continuity of the temperature carrier at the liquid-wall boundary [4].

Analysis of recent research and publications. Non-isothermal conditions of technological processes in chemical-technological equipment are much more common than isothermal ones. Today, there are many methods of supply and removal of heat flows to or from the heat transfer surface of machines and apparatus: including the “pipe in pipe” principle or those with an intermediate shell. The magnitude of the heat flow through a solid surface is determined by its thermal resistance and the heat transfer coefficients from the sides of the heat exchanging media [5]. If the medium is Newtonian, then the heat transfer coefficients are determined using known formulas [6]. As mentioned above, much less is studied in the non-Newtonian medium [1]. In this work the authors study the calculation of the coefficients of heat transfer during the flow of non-Newtonian fluid in a pipe or channel of chemical and technological equipment. From the analysis of the technical literature, we can conclude that among the variety of non-Newtonian fluids, the most common are three classes: Bingam fluids, generalized displaced fluids and power fluids. Under the term "generalized-displaced fluids" we mean liquids whose viscosity depends on the shear rate in an arbitrary manner. A special case of such fluids is power fluid [7]. Flow sections such as pipes and ducts are chosen because the pipe is the main element of the heat exchangers and the duct is the main element of the working chamber of the worm extruder [8]. The content of this work is based on a number of results on the flow of the fluids mentioned above in pipes and ducts [9, 10, 11, 12, 13, 14]. In particular, [14, 15] considered the flow in channels of flat and rectangular shapes whose boundaries move along themselves, as well as in longitudinal and transverse directions. In [14, 15, 16], three-dimensional fields of non-Newtonian fluid flow were constructed at different boundary conditions, which form the necessary conditions for calculating the heat transfer coefficients. As it is known, the flow of fluid in a pipe or duct can be organized so that in the process of supply or removal of heat the thermal boundary layer is formed (or not formed) [17]. The flow of the fluid itself also may (or may not) form a hydrodynamic boundary layer. The flow in which the hydrodynamic layer is absent or, in the same way, occupies the entire cross-section of a pipe or channel is called stabilized [18]. Otherwise, it is unstable [18]. The same is true of the temperature transfer process [19]. The measure of the ratio of the thickness of the hydrodynamic and thermal boundary layers is the Prandtl number [18, 19]. For most of the flows, the thickness of the hydrodynamic boundary layer is greater than the thermal one, and the Prandtl number is greater than one. This is especially true if the hydrodynamic boundary layer occupies the entire cross section of a pipe or channel [18, 19].

Results of the research and their discussion

This paper deals with stabilized flows of non-Newtonian fluids in a hydrodynamic sense and with destabilized temperature transfer with respect to the thermal boundary layer. The latter condition means that the Péclet number is much greater than one [20]. The convective temperature transfer equation is used to calculate the heat transfer coefficients [1,2,3]. The heat transfer is affected by the velocity component, which can be both tangent and normal relative to the heat transfer surface [1]; in straight channels and pipes with a stable flow, the normal velocity component is absent. A tangential velocity component can have two components - along and across the longitudinal axis of a pipe or channel. In this case, the tangent velocity component is the vector sum of these components, and it is this sum that determines the heat transfer coefficient.

The equation of convective temperature transfer is written as follows:

$$\upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y} = \chi \frac{\partial^2 T}{\partial y^2}, \quad \chi = \frac{\lambda}{\rho c_p} \tag{1}$$

where υ_x and υ_y – tangent and normal components of non-Newtonian fluid velocity vector, m/s; T – absolute temperature of the liquid, K; χ – temperature conductivity of the liquid, m²/s; λ – thermal conductivity of the liquid, W/m·K; ρ – fluid density, kg/ m³; c_p – heat capacity of the liquid, J/kg·K. Equation (1) is written in the approximation of the thermal boundary layer so that only the transverse derivative of the variable y is preserved in its right side. The x coordinate is considered to be aligned along the tangent component of the velocity of the fluid (in the case of purely longitudinal flow, the tangent component is directed along the axis of the pipe or channel). The above is shown in Fig. 1.

Consider $\upsilon_y \equiv 0$, given the fact that there is a thermal boundary layer and the heat flux near the solid surface depends on the behavior of the velocity field only near that surface. The second and first order decompositions for Bingam and non-Bingam fluids, respectively, should be used according to the small distance to the solid surface. If this distance is denoted as, the $\tilde{y}$ following expression should be used for Bingam and non-Bingam fluids near the boundaries of sections of the flow (channel pipe walls):

$$\upsilon_x = w_\Gamma + \frac{\partial \upsilon_x}{\partial \tilde{y}} \tilde{y}, \tag{2}$$

Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

while for Bingam fluid near a solid core, the following expression should be used:

$$\upsilon_x = \upsilon_K + \frac{\partial^2 \upsilon_x}{\partial \tilde{y}} \tilde{y}^2 , \qquad (3)$$

where w_Γ - velocity of the flow near wall, m/s; υ_k - velocity of the solid core, m/s. In (3) addition, proportional to $\tilde{y}$ is absent because the second invariant of the deformation rate tensor turns to zero [7].

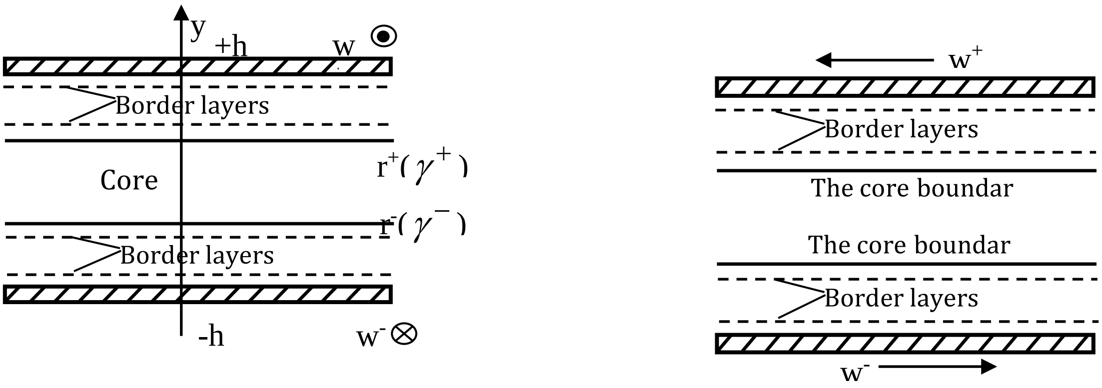


Fig. 1. Thermal boundary layers in Bingam fluid: a - cross-sectional view; b - view along a flat channel

Equations (1) from υ_x to (2) and (3) allows self-driving solutions with the help of substitutions of this following kind [17]:

$$\omega = \left(\frac{2}{\chi} \frac{\partial \upsilon_x}{\partial \tilde{y}} \right)^{1/3} \frac{\tilde{y}}{x^{1/3}} .$$

(for υ_x according to formula (2)); (4)

$$\omega = \left(\frac{2}{\chi} \frac{\partial^2 \upsilon_x}{\partial \tilde{y}^2} \right)^{1/4} \frac{\tilde{y}}{x^{1/4}} .$$

(for υ_x according to formula (3))

as see from (4), the heat flow density decreases along the direction of the tangent velocity $x^{-1/3}$ and $x^{-1/4}$ for first and second cases respectively. By entering the average value of the heat flow density at some length L and considering the standard definition of the Nusselt number for the latter, we obtain the following expressions:

$$\mathrm{Nu} = \frac{2}{3} 2^{2/3} \frac{h}{L^{1/3}} \left(\frac{1}{\chi} \frac{\partial \upsilon_x}{\partial \tilde{y}} \right)^{1/3} ,$$

$$\mathrm{Nu} = \frac{8}{15} \frac{h}{L^{1/4}} \left(\frac{1}{\chi} \frac{\partial \upsilon_x}{\partial \tilde{y}} \right)^{1/4} , \qquad (5)$$

in which h – half-width of the channel, pipe, m. Thus, it follows from (5) that the Nusselt numbers are determined by the derivatives of the velocity tangent to the wall on the walls of channels, pipes and at the boundaries of the solid core for the Bingam fluid. Below are the flows of bingam fluid in the flat and rectangular channels (see Figs. 2 and 3).

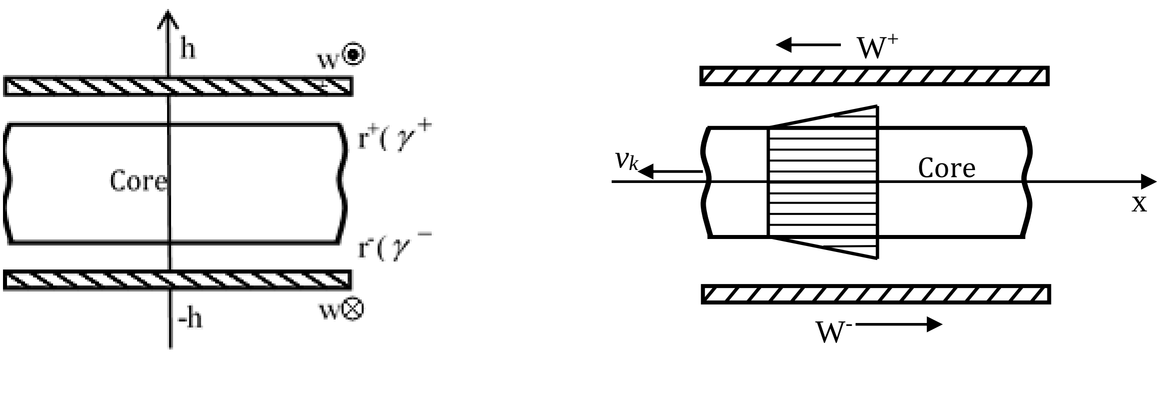


Fig. 2. Longitudinal flow of bingam fluid in a flat channel: a - cross-sectional view; b - view along the channel

Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

The velocity of the longitudinal flow in a flat channel is as follows [7]

$$\upsilon_{\|} = w_{\|}^{\pm} - \frac{h}{2\mu}\frac{dP}{d\zeta_h}\left(1-\xi_y^2\right) \pm \frac{\gamma^{\pm}}{\mu}\frac{dP}{d\zeta_h}\left(1 \mp \xi_y\right)$$

$$\xi_y = y/h; \qquad (6)$$

$$\gamma^{\pm} = \pm\frac{\tau_0}{dP/d\zeta_h} + \frac{\left(w_{\|}^{+} - w_{\|}^{-}\right)/2}{\frac{1}{\mu}\frac{dP}{d\zeta_h} - \frac{\tau_0}{\mu}},$$

$$\xi_h = x/h, \quad \mu_{ef} = \mu + \frac{\tau_0}{\sqrt{I_2}},$$

where $\upsilon_{\|}$ – velocity of the flow, m/s; μ – the viscosity of the Bingam fluid, Pa s; P – pressure in the Bingam fluid, Pa; $w_{\|}^{\pm}$ – velocities of the upper and lower boundaries of the channel, respectively, m/s (see Figure 2); $\gamma^{\pm}$ – dimensionless coordinates of the solid core; τ_0 – the boundary of the flow, Pa.

The velocity of the longitudinal flow of bingam fluid in a rectangular channel is written as follows [14; 15]:

$$\upsilon_{\|y}^{\pm} = -\frac{h}{2\mu}\frac{dP}{d\zeta_h}\cdot\frac{1-\xi_y^2}{1+æ^2\rho_y^{\pm}\left(1\mp\gamma_y^{\pm}\right)} \pm \frac{h}{\mu}\frac{dP}{d\zeta_h}\cdot\frac{\gamma_y^{\pm}\cdot\left(1\mp\xi_y\right)}{1+æ^2\rho_y^{\pm}\left(1\mp\gamma_y^{\pm}\right)} + w_{\|y}^{\pm}, \qquad (7)$$

$$\upsilon_{\|x}^{\pm} = -\frac{a}{2\mu}\frac{dP}{d\zeta_a}\cdot\frac{1-\xi_x^2}{1+\rho_x^{\pm}\left(1\mp\gamma_x^{\pm}\right)/æ^2} \pm \frac{a}{2\mu}\frac{dP}{d\zeta_a}\cdot\frac{\gamma_x^{\pm}\cdot\left(1\mp\xi_x\right)}{1+\rho_x^{\pm}\left(1\mp\gamma_x^{\pm}\right)/æ^2} + w_{\|x}^{\pm},$$

$$æ = h/a; \quad \xi_y = y/h; \quad \xi_x = x/a; \quad \xi_h = z/h; \quad \xi_a = z/a.$$

where *a* - the width of the rectangular channel, m; *h* - the height of this channel, m; $\gamma_y^{\pm}$ and $\gamma_x^{\pm}$ - dimensionless boundaries of the solid core. The values for $\gamma_x^{\pm}$ and $\gamma_y^{\pm}$ are determined by the authors [14; 15]. Their precice view is the linear combination of expressions (6) for $\gamma^{\pm}$ with weight factors, dependent on the channel shape parameter æ [14; 15].

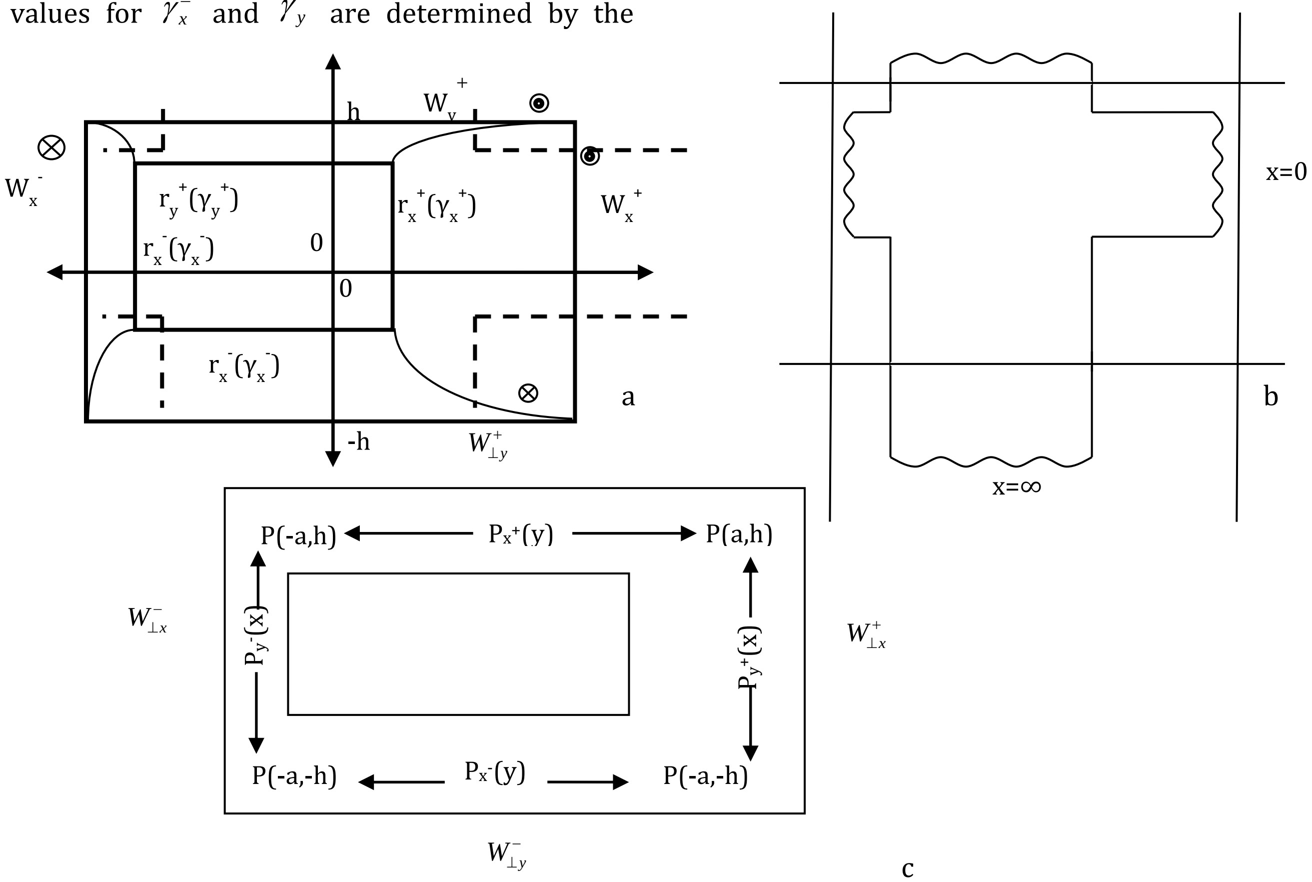


Fig. 3. Longitudinal and transverse flow of bingam fluid in a rectangular channel: a - longitudinal flow of bingam fluid in a rectangular channel; b - longitudinal flow of bingam fluid in a rectangular channel, as a composition of flat liquids; c - transverse flow of bingam fluid in a rectangular channel

Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

The velocity of the longitudinal and transverse flows of bingam fluid in a rectangular channel with velocity boundaries (see Fig. 3) for the longitudinal velocity component coincides with the expressions (7), and the transverse velocity components $\upsilon^{\pm}_{\perp y}$ and $\upsilon^{\pm}_{\perp x}$ are represented by the following expressions [7]:

$$\upsilon^{\pm}_{\perp y}=\frac{h}{2\mu}\frac{dP_y}{d\theta_h}\cdot\frac{\xi_y^2-\gamma_y^{\pm 2}\pm\left(1\pm\gamma_y^{\pm}\right)\left(\gamma_y^{\pm}-\xi_y\right)}{1+æ^2 r^{\pm}\cdot\left(1\mp\gamma_y^{\pm}\right)}\pm\frac{w^{\pm}_{\perp y}\cdot\left(\xi_y-\gamma_y^{\pm}\right)}{1\mp\gamma_y^{\pm}}, \quad (8)$$

$$\upsilon^{\pm}_{\perp x}=\frac{a}{2\mu}\frac{dP_x}{d\theta_a}\cdot\frac{\xi_x^2-\gamma_x^{\pm 2}\pm\left(1\pm\gamma_x^{\pm}\right)\left(\gamma_x^{\pm}-\xi_x\right)}{1+s^{\pm}\cdot\left(1\mp\gamma_x^{\pm}\right)/æ^2}\pm\frac{w^{\pm}_{\perp x}\cdot\left(\xi_x-\gamma_x^{\pm}\right)}{1\mp\gamma_x^{\pm}},$$

$$\theta_h=x/h;\qquad \xi_y=y/h;$$
$$\theta_y=y/a;\qquad \xi_x=x/a;$$

where $dP_y/d\theta_h$, $dP_x/d\theta_a$ - transverse pressure gradients in the cross-sectional plane of the channel, Pa/m. (see Fig. 3).

УIn formulas (7) and (8) there are valuies $\rho_y^{\pm}$, $\rho_x^{\pm}$, $r^{\pm}$, $s^{\pm}$, which depend on the combinations of boundary conditions in fractional and rational ways, which are not presented here due to the complexity of the calculations [7]. Based on formulas (4) and (5), the corresponding derivatives should be calculated. For the flat channel and the longitudinal flow there are two coefficients of heat transfer - on the upper and lower walls; and two heat transfer coefficients, at the upper and lower bounds of the solid core. This means that the first derivatives of (6) in points $\xi_y=\pm 1$ and the second derivatives of the same expression in points $\xi_y=\gamma^{\pm}$ must be calculated. By omitting simple intermediate actions, you can write the following result:

$$\left.\frac{\partial\upsilon_{\|}^{\pm}}{\partial\tilde{y}}\right|\pm 1=\frac{1}{\mu}\frac{dP}{d\zeta_h}\left(1\mp\gamma^{\pm}\right);\qquad \left.\frac{\partial^2\upsilon_{\|}^{\pm}}{\partial\tilde{y}^2}\right|=\frac{1}{\mu h}\frac{dP}{d\zeta_h} \quad (9)$$

Therefore, Nusselt numbers are proportional to the cubic root of the first expression and the fourth degree root of the second expression (9). Taking into account expression (6) for $\gamma^{\pm}$, it turns out that Nusselt numbers in a rather complicated way depend on all the parameters of the flow: pressure gradient, flow threshold, velocity differences of the flow near walls.

In the longitudinal flow of Bingam fluid in a rectangular channel there are eight boundaries - four walls and four boundaries of the solid core. Therefore, there are eight heat transfer coefficients. To calculate them, it is necessary to determine four first derivtives of expressions (7) by y and x respectively in points $\xi_y=\pm 1$; $\xi_x=\pm 1$ and four second derivatives by y and x respectively in points $\xi_y=\gamma_y^{\pm}$, $\xi_x=\gamma_x^{\pm}$. By omitting the intermediate transformations, the final result can be written as follows:

$$\left.\frac{\partial\upsilon^{\pm}_{\|y}}{\partial\tilde{y}}\right|\pm 1=\frac{1}{\mu}\frac{dP}{d\zeta_h}\cdot\frac{\left(1\mp\gamma_y^{\pm}\right)}{1+x^2\rho_y^{\pm}\cdot\left(1\mp\gamma_y^{\pm}\right)};\quad \left.\frac{\partial^2\upsilon^{\pm}_{\|y}}{\partial\tilde{y}^2}\right|_{\gamma_y^{\pm}}=\frac{1}{\mu h}\frac{dP}{d\zeta_h}\cdot\frac{1}{1+x^2\rho_y^{\pm}\cdot\left(1\mp\gamma_y^{\pm}\right)}$$

$$\left.\frac{\partial\upsilon^{\pm}_{\|x}}{\partial\tilde{x}}\right|\pm 1=\frac{1}{\mu}\frac{dP}{d\zeta_a}\cdot\frac{\left(1\mp\gamma_x^{\pm}\right)}{1+x^2\rho_x^{\pm}\cdot\left(1\mp\gamma_x^{\pm}\right)/x^2}; \quad (10)$$

$$\left.\frac{\partial^2\upsilon^{\pm}_{\|x}}{\partial\tilde{x}^2}\right|_{\gamma_x^{\pm}}=\frac{1}{\mu a}\frac{dP}{d\zeta_a}\cdot\frac{1}{1+\rho_x^{\pm}\cdot\left(1\mp\gamma_x^{\pm}\right)/x^2}$$

Finally, in the longitudinal-transverse flow in the rectangular channel there are eight coefficients of mass transfer. The calculation is exactly the same as the calculation for the longitudinal flow. Formulas (7) and (8) should be used. The only difference from the longitudinal-transverse flow is that instead of $\upsilon^{\pm}_{\|y}$ and $\upsilon^{\pm}_{\|x}$ we should calculate the first an second derivatives from $\sqrt{\left(\upsilon^{\pm}_{\|y}\right)^2+\left(\upsilon^{\pm}_{\perp y}\right)^2}$ and $\sqrt{\left(\upsilon^{\pm}_{\|x}\right)^2+\left(\upsilon^{\pm}_{\perp x}\right)^2}$. An example below is a calculation for speed with index «y». The following expressions are relevant:

$$\frac{\partial\upsilon_y^\pm}{\partial\tilde{y}}=\frac{1}{2\sqrt{\left(\upsilon_{\|y}^\pm\right)^2+\left(\upsilon_{\perp y}^\pm\right)^2}}\cdot\left\{2\upsilon_{\|y}^\pm\frac{\partial\upsilon_{\|y}^\pm}{\partial\tilde{y}}+2\upsilon_{\perp y}^\pm\frac{\partial\upsilon_{\perp y}^\pm}{\partial\tilde{y}}\right\},\quad \upsilon_y^\pm\equiv\sqrt{\left(\upsilon_{\|y}^\pm\right)^2+\left(\upsilon_{\perp y}^\pm\right)^2}, \qquad (11)$$

$$\frac{\partial^2\upsilon_y^\pm}{\partial\tilde{y}^2}=-\frac{1}{2}\frac{1}{\left(\left(\upsilon_{\|y}^\pm\right)^2+\left(\upsilon_{\perp y}^\pm\right)^2\right)^{3/2}}\cdot\left\{2\upsilon_{\|y}^\pm\frac{\partial\upsilon_{\|y}^\pm}{\partial y}+2\upsilon_{\perp y}^\pm\frac{\partial\upsilon_{\perp y}^\pm}{\partial y}\right\}^2+\frac{1}{2\sqrt{\left(\upsilon_{\|y}^\pm\right)^2+\left(\upsilon_{\perp y}^\pm\right)^2}}\times$$

$$\times\left\{2\left(\frac{\partial\upsilon_{\|y}^\pm}{\partial y}\right)^2+2\left(\frac{\partial\upsilon_{\perp y}^\pm}{\partial y}\right)^2+2\upsilon_{\|y}^\pm\frac{\partial\upsilon_{\|y}^\pm}{\partial y}+2\upsilon_{\perp y}^\pm\frac{\partial\upsilon_{\perp y}^\pm}{\partial y}\right\}.$$

If expressions (11) are taken in points $\xi_y=\pm1$ and $\xi_y=\gamma_y^\pm$, then (11) are simplified, so we receive the following results:

$$\left.\frac{\partial\upsilon_y^\pm}{\partial\tilde{y}}\right|_{\pm1}=\frac{1}{2\sqrt{\left(w_{\|y}^\pm\right)^2+\left(w_{\perp y}^\pm\right)^2}}\cdot\left(2w_{\|y}^\pm\cdot\left.\frac{\partial\upsilon_{\|y}^\pm}{\partial y}\right|_{\pm1}+2w_{\perp y}^\pm\cdot\left.\frac{\partial\upsilon_{\perp y}^\pm}{\partial y}\right|_{\pm1}\right),$$

$$\left.\frac{\partial^2\upsilon_к^\pm}{\partial\tilde{y}^2}\right|_{\gamma_{\tilde{y}}^\pm}=2\upsilon_k\left(\left.\frac{\partial^2\upsilon_{\|y}^\pm}{\partial y^2}\right|_{\gamma_{\tilde{y}}^\pm}+\left.\frac{\partial^2\upsilon_{\perp y}^\pm}{\partial y^2}\right|_{\gamma_{\tilde{y}}^\pm}\right), \qquad (12)$$

ijn which υ_k - velicoty of the colid core, m/s. This value is defined in the work of the authors [7] and due to the complexity of the calculations it is not given here.

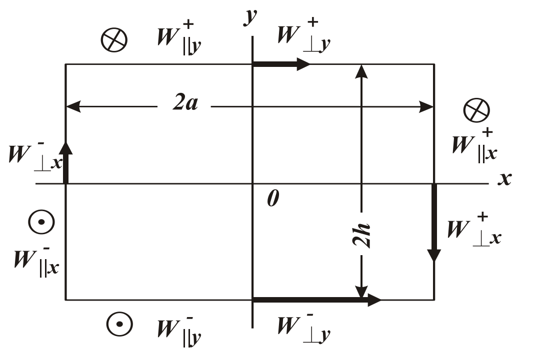


Fig. 4. Rectangular channel and boundary conditions of three-dimensional flow in the channel: $W_{IIY}^\pm$ - value of longitudinal velocity on the walls of the channel, normal to the OY axis; $W_{IIX}^\pm$ - the value of the longitudinal velocity on the walls of the channel, normal to the OX axis; $W_{\perp X}^\pm, W_{\perp Y}^\pm$ - values of the transverse velocities at the channel boundaries.

Derivatives from components $\upsilon_x^\pm$ by $\tilde{x}$ are calculated in the same way. The result coincides with (12), taking into account the replacement of index «y» with index «x» and values $w_{\|y}^\pm$, $w_{\perp y}^\pm$ with values $w_{\|x}^\pm$ i $w_{\perp x}^\pm$ (see Fig. 3). The very first and second derivatives included in (11) and (12) are calculated from expressions (8).

According to the written above for the Bingam fluid, the calculation of the heat transfer coefficients for the flow of generalized and power fluids should be considered. Below, we consider the longitudinal flow in a flat channel as the base flow, similar to the same flow of Bingam fluid. Then the longitudinal and transverse flow in the flat channel and the longitudinal flow in the rectangular channel are considered (see Figs. 4-7).The boundary conditions for the basic problem of the Couett flow are shown in Fig. 5.

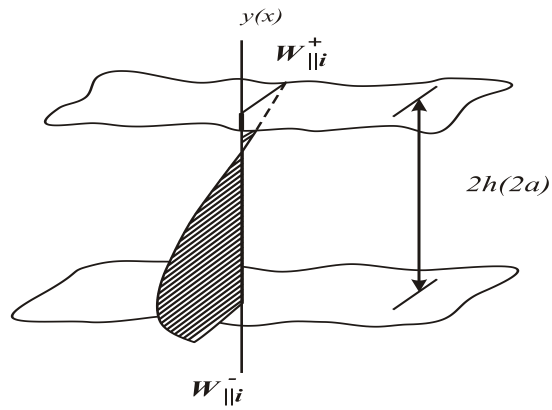


Fig. 5. Slit channel fragment and boundary conditions: $i=x,y$; $i=x$, width of channel – 2*a*; $i=y$, width of channel 2*h*

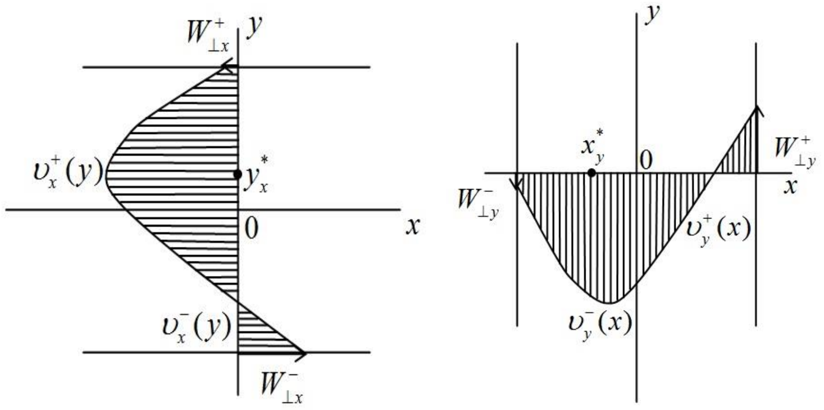
The expression for a longitudinal flow profile in a flat channel is as follows [7]:

$$\upsilon_z^\pm=\frac{\alpha\left(h\mp y\right)}{\beta}+\left(\frac{\alpha^2}{4\beta^2}\pm\frac{y^*-y}{\beta}\frac{dP}{dz}\right)^{3/2}\frac{2}{3}\frac{\beta}{dP/dz}-\left(\frac{\alpha^2}{4\beta^2}+\frac{h\mp y^*}{\beta}\frac{dP}{dz}\right)\frac{2}{3}\frac{\beta}{dP/dz}+w^\pm,$$

$$y^*=\frac{w^+-w^-}{\dfrac{\alpha}{\beta}-2\left(\dfrac{\alpha^2}{4\beta^2}+\dfrac{h}{\beta}\dfrac{dP}{dz}\right)^{1/2}};\qquad \mu_{ef}=\alpha+\beta\sqrt{I_2}, \qquad (13)$$

Journal of Chemistry and Technologies, 2020, *28*(1), 88-99

in which μ_{ef} – the viscosity of the generalized fluid Pa·s; α, β – viscosity parameters; W^+ and W^- – movement velocity of the flow near channel walls, m/s. (see Figure 4); z – coordinate along the axis of the channel, m. The expressions for the velocity of a longitudinal transverse flow in a flat channel in the longitudinal direction have the form similar to (13), but due to the fact that there are two velocity components in this flow, the value β varies by this rule: $\beta \to \beta_z = \beta\sqrt{(1+k^2)/2}$, where value k depends on the velocities at the boundaries of the channel in the following way [14].



a b

Fig. 6. Transverse flow in the slit channel: a - depending on the coordinate *y*, b - depending on the coordinate *x*

$$k = \frac{w_\perp^+ - w_\perp^-}{w_\parallel^+ - w_\parallel^-} \tag{14}$$

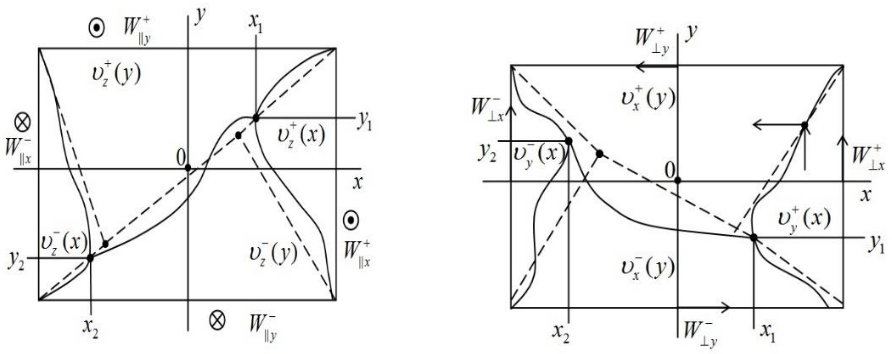
where $w_\perp^\pm$, $w_\parallel^\pm$ - velocities of the of the flow near channel walls in the longitudinal and transverse directions, respectively. The value of α in the longitudinal transverse flow coincides with the same value of the longitudinal flow. Transverce velocity component υ_y^+ is also described by the formula (13), вin which the longitudinal pressure gradient dP/dz must be replaced with a transverse gradient dP/dy; and value β should be replaced with value β_x according to the rule: $\beta \to \beta_x = \beta\sqrt{(1+k^2)/2k^2}$. The value of α remains equal to its value in the longitudinal flow. The longitudinal transverse flow is characterized by two special points y_z^* and y_y^*, expressions for which are derived from expression (13) for coordinate y^* according to the rules:

$$y_z^*\left(w^+ - w^-, \alpha, \beta, dP/dz\right) \to y_z^*\left(w_\parallel^+ - w_\parallel^-, \alpha, \beta_z, dP/dz\right);$$

$$y_y^*\left(w^+ - w^-, \alpha, \beta, dP/dz\right) \to y_y^*\left(w_\perp^+ - w_\perp^-, \alpha, \beta_x, dP/dx\right);$$

Value dP/dx is calculated from the following formula [7]:

$$\frac{dP}{dx} = \frac{3}{2}\alpha\frac{w_\perp^+ + w_\perp^-}{h^2}\cdot\left(\frac{\alpha}{\alpha+\beta_x}\right)^m + \frac{7}{2}\frac{\beta_x}{h^3}\left(w_\perp^+ - w_\perp^-\right)^2\left(\frac{\beta_x}{\alpha+\beta_x}\right)^n, \tag{15}$$



a b

Fig. 7. Rectangular channel breakdown and breakdown Linearization:
a – for longitudinal flow; b – for transverse flow

A longitudinal flow in a rectangular channel is also constructed from velocity profiles (13) as the main parameters. This is done by splitting the rectangular cross-section of the channel into subdomains in which the longitudinal velocity depends on the *x* and *y* coordinates separately [14]. The compact expression of the expression for longitudinal velocity is as follows:

$$\upsilon_{zi}^{\pm}=\frac{\alpha_i}{2\beta_i}\left(l_i\mp x_i\right)+\left(\frac{\alpha_i^2}{4\beta_i^2}\pm\frac{x_i x_i^*}{\beta_i}\frac{dP}{dz}\right)^{3/2}\frac{2}{3}\frac{\beta_i}{dP/dz}-\left(\frac{\alpha_i^2}{4\beta_i^2}+\frac{\alpha_i\mp x_i^*}{\beta_i}\frac{dP}{dz}\right)^{3/2}\frac{2}{3}\frac{\beta_i}{dP/dz};$$

$$x_i^*=\frac{w_i^+-w_i^-}{\dfrac{\alpha_i}{\beta_i}-2\left(\dfrac{\alpha_i^2}{4\beta_i^2}+\dfrac{l_i}{\beta_i}\dfrac{dP}{dz}\right)^{1/2}};\quad i=y,x;\quad l_i=h,a;\quad x=h/a; \tag{16}$$

$$\alpha_y=\alpha\left(1+æ^2\right);\qquad \beta_y=\beta\left(1+æ^2\right)^{3/2}/2;$$

$$\alpha_x=\alpha\left(1+1/æ^2\right);\qquad \beta_x=\beta\left(1+1/æ^2\right)^{3/2}/2,$$

where a – the width of the rectangular channel, m.

When calculating the heat transfer coefficients of the flows whose velocities are represented by formulas (13), (14), (15), and (16), it is necessary to use formulas (5), while taking into account that in a plane channel in the longitudinal and longitudinal transverse flows there two coefficients of heat transfer, and in a rectangular channel — there are four coefficients of heat transfer. In a longitudinal transverse flow in a flat channel, it is necessary to add two components of tangent velocity according to Pythagorean theory. As for the Bingam fluid flow, the calculation of the heat transfer coefficients is reduced to calculating the first velocity derivatives near the channel boundaries. For a longitudinal flow in a flat channel, the values of the derivatives in points $y=\pm h$ are equal to:

$$\left.\frac{\partial\upsilon^{\pm}}{\partial y}\right|_{y=\pm h}=\mp\left(\frac{\alpha^2}{4\beta^2}\pm\frac{h\mp y^*}{\beta}\frac{dP}{dz}\right)^{1/2}. \tag{17}$$

For the longitudinal transverse flow, the rule of finding the vector module (see formula (17) of the expression for $\upsilon_y^{\pm}$) must be used in the calculation of the derivatives. As a result of calculations for derivatives, the following expressions are correct:

$$\left.\frac{\partial\tilde{\upsilon}^{\pm}}{\partial y}\right|_{y=\pm h}=\frac{1}{2\sqrt{\left(w_{\parallel}^{\pm}\right)^2+\left(w_{\perp}^{\pm}\right)^2}}\left\{2w_{\parallel}^{\pm}\left.\frac{\partial\upsilon_z^{\pm}}{\partial y}\right|_{y=\pm h}+2w_{\perp}^{\pm}\left.\frac{\partial\upsilon_x^{\pm}}{\partial y}\right|_{y=\pm h}\right\},$$

$$\left.\frac{\partial\tilde{\upsilon}_z^{\pm}}{\partial y}\right|_{y=\pm h}=\mp\frac{\alpha}{2\beta_z}\pm\left(\frac{\alpha^2}{4\beta_z^2}+\frac{h\mp y_z^*}{\beta_z}\frac{dP}{dz}\right)^{1/2};\qquad \beta_z=\beta\sqrt{\left(1+k^2\right)/2};$$

$$\left.\frac{\partial\tilde{\upsilon}_x^{\pm}}{\partial y}\right|_{y=\pm h}=-\frac{\alpha}{2\beta_x}\pm\left(\frac{\alpha^2}{4\beta_x^2}+\frac{h\mp y_x^*}{\beta_x}\frac{dP}{dz}\right)^{1/2};\qquad \beta_x=\beta\sqrt{\left(1+k^2\right)/2k^2}.$$

$$y_z^*=\frac{w_{\parallel}^+-w_{\parallel}^-}{\dfrac{\alpha}{\beta_z}-2\left(\dfrac{\alpha^2}{4\beta_z^2}+\dfrac{h}{\beta_z}\dfrac{dP}{dz}\right)^{1/2}}; \tag{18}$$

$$y_x^*=\frac{w_{\perp}^+-w_{\perp}^-}{\dfrac{\alpha}{\beta_x}-2\left(\dfrac{\alpha^2}{4\beta_x^2}+\dfrac{h}{\beta_x}\dfrac{dP}{dx}\right)^{1/2}}.$$

.

The calculation of derivatives for longitudinal flow in a rectangular channel is performed according to the formulas (16) and is reduced to formulas of type (17) with corresponding values α_i, β_i, x_i^*.

It makes sense to separately consider the calculation of heat transfer coefficients for the flow of a power fluid. Due to the fact that a number of coolants is characterized by the viscosity of a power fluid [15]. Due to the fact that the main points of the calculations are exactly similar to those described for Bingam and generalized liquids, only the longitudinal flow of a power fluid in a flat channel is considered below. The expression for the velocity of the longitudinal flow of a power fluid with exponent is as follows:

$$\upsilon^{\pm}=\left|\frac{y-y^{*}}{\beta}\cdot\frac{dP}{dz}\right|^{\frac{n+2}{n+1}}\cdot\frac{n+1}{n+2}\frac{\beta}{dP/dz}-\left|\frac{h\mp y^{*}}{\beta}\frac{dP}{dz}\right|^{\frac{n+2}{n+1}}\cdot\frac{n+1}{n+2}\frac{\beta}{dP/dz}+w^{\pm},$$

$$y^{*}=\frac{w^{+}-w^{-}}{2\left(\frac{h}{\beta}\frac{dP}{dz}\right)^{\frac{1}{n+1}}},\qquad \mu=\beta\left|\frac{d\upsilon^{\pm}}{dy}\right|^{n}. \quad (19)$$

The calculation of the derivative of (19) for the heat transfer coefficient results in the following result:

$$\left.\frac{\partial\upsilon^{\pm}}{\partial y}\right|_{y=\pm h}=\left(\frac{h\mp y^{*}}{\beta}\frac{dP}{dz}\right)^{\frac{1}{n+1}}. \quad (20)$$

The presented resultss indicate that the dependency of the Nusselt number on the pressure gradients (longitudinal and transverse) of the rheological and geometric characteristics of liquids and channels is nonlinear and very complex. In order to make this dependency simpler and clearer, you should write down the expressions for the Nusselt number for the simplest flow, that is, for the longitudinal flow in a flat channel. The more complex flows also have Nusselt numbers, but the numbers are more quantitative than the complicated ones.

The expressions for the Nusselt numbers below are written according to formulas (5) up to the trivial factors at velocity derivatives. For Bingam fluid, the Nusselt number is proportional to the following expression:

$$\mathrm{Nu}\sim\left\{\frac{1}{\mu}\frac{dP}{d\zeta}\left[1\,m\frac{\tau_o}{dP/d\zeta}-\frac{\left(w^{+}-w^{-}\right)/2h}{dP/d\zeta\times(1/\mu)\times(1-\tau_o/dP/d\zeta)}\right]\right\}^{1/3}, \text{(on the walls)}$$

$$\mathrm{Nu}\sim\left(\frac{1}{\mu}\frac{dP}{d\zeta}\right)^{1/4}, \text{(at the boundaries of the solid nucleus)} \quad (21)$$

From this expression it follows that the first two Nusselt numbers depend on three parameters: $(1/\mu)(dP/d\zeta)$; $\tau_o/(dP/dz)$; $\left(w^{+}-w^{-}\right)/2h$, the last of which is the kinematic velocity of the displacement of the Couett flow of the Newtonian fluid.

For a generalized fluid, the correct expression for Nusselt numbers is the following:

$$\mathrm{Nu}\sim\left\{\pm\frac{\alpha}{2\beta}\pm\left[\frac{\alpha^{2}}{4\beta^{2}}\pm\frac{1}{\beta}\frac{dP}{d\zeta}\cdot\left(1\mp\frac{\left(w^{+}-w^{-}\right)/2h}{\frac{\alpha}{2\beta}-\left(\frac{\alpha^{2}}{4\beta^{2}}+\frac{1}{\beta}\frac{dP}{d\zeta}\right)^{1/2}}\right)\right]^{1/2}\right\}^{1/3}, \quad (22)$$

From which it can be seen that the Nusselt number also depends on the following three parameters: $\alpha/2\beta$; $(1/\beta)(dP/d\zeta)$; $\left(w^{+}-w^{-}\right)/2h$.

The power fluid has a Nusselt number proportional to the following expression:

$$\mathrm{Nu}\sim\left\{\frac{1}{\beta}\frac{dP}{d\zeta}\left[1\mp\frac{\left(w^{+}-w^{-}\right)/h}{\left(\frac{1}{\beta}\frac{dP}{d\zeta}\right)^{\frac{1}{n+1}}}\right]\right\}^{\frac{1}{3(n+1)}} \quad (23)$$

which includes two parameters: $(1/\beta)(dP/d\zeta)$ and $\left(w^{+}-w^{-}\right)/2h$.

Conclusion

Based on the above, we can draw the following conclusions. All presented results refer to hydrodynamically stabilized flows. Nusselt numbers for Bingam fluid and other non-Newtonian fluids at the walls of the channels are determined by the first derivative of the velocity aligned along the normal to the wall. Nusselt numbers for the Bingam fluid within the solid core are determined by the second derivative of the velocity for to the normals to the boundary. If the tangent velocity of the fluid flow at the walls has two components, then the velocity, the derivative of which is included in the Nusselt number, is determined through these components according to Pythagoras' theorem. Nusselt numbers for Bingam and generalized fluids depend on three parameters for longitudinal flow in a flat channel. If a flow is more complex, that is, two or three-dimensional, then the number of parameters increases so that these parameters are generated by each velocity component of the multidimensional flow and form all possible combinations.

Subsequently, the value of Nusselt numbers allows us to calculate the corresponding coefficients of heat transfer and heat return between non-Newtonian fluids, pipes and channels, and the environment.

Bibliography

[1] Харламов С. Гидродинамика и теплообмен: Новые тенденции и перспективы моделирования внутренних течений / С. Харламов – LAP LAMBERT Academic Publishing, 2013. – 116 с.

[2] Ульев Л .М. Ламинарное течение в соосных конических каналах / Л .М. Ульев – Харьков: НТУ «ХПИ», 2006 – 660 с.

[3] Cantelli A. Uniform Flow of Modified Bingham Fluids in Narrow Cross Sections / A. Cantelli // Journal of Hydraulic Engineering, 2009. – 135(8). – P. 640–650.

[4] Baptista A. Heat transfer in fully developed laminar flow of power law fluids / A. Baptista, M.A. Alves, P.M. Coelho // Journal of Heat Transfer, 2014. – 136(4). – P. 1–8.

[5] Голованчиков А. Б. Моделирование течения вязкопластической реакционной массы в шнековом реакторе с маловязким пристенным слоем / Голованчиков А. Б., Шагарова А. А. // Известия вузов. Химия и химическая технология. – 2015. – Т. 58, № 12. – С. 69–72.

[6] Пономарева М. А. Циркуляционное течение высоковязкой неньютоновской жидкости в канале одношнекового экструдера / М. А Пономарева, М. П. Филина, В. А. Якутенок // Вестник Томского государственного университета. Математика и механика. 2016. – № 2(40). – С. 97–107.

[7] Товажнянський Л. Л. Моделювання течій неньютонівських рідин у каналах базової геометрії / Л. Л. Товажнянський, Е.В. Білецький, Ю. А.Толчинський – Харьков: НТУ «ХПІ», 2013 – 319 с.

[8] Колгрюбер К. Двухшнековые сонаправленные экструдеры. Основы, технология, применение / К. Колгрюбер – СПб.:.: Профессия, 2016. – 352 с.

[9] Рябинин М. В. Методика определения потерь на трение в гидравлически гладкой круглой трубе для псевдопластичных жидкостей / М. В. Рябинин, К. А. Труханов // Современные проблемы науки и образования. 2015. – № 1-1. – [Электронный ресурс]. – Режим доступа: http://www.science-education.ru

[10] M. Anandha Rao. Rheology of Fluid, Semisolid, and Solid Foods: Principles and Applications / Rao M. Anandha - New York: Imprint: Springer, 2014. – 461 p.

[11] Перминов А. В. Устойчивость стационарного плоскопараллельного течения псевдопластической жидкости в плоском вертикальном слое / А. В. Перминов, Т. П. Любимова // Вычислительная механика сплошных сред. – 2014. – Т. 7, № 3. – С. 270 – 278.

[12] Mitsoulis E. Numerical simulations of complex yield-stress fluid flows / E. Mitsoulis, J. Tsamopoulos // Rheologica Acta 56. – 2017. – P. 231–258.

[13] Frigaard I. A. Binghams model in the oil and gas industry / I. A. Frigaard, K. Paso, P. de Souza Mendes // Rheologica Acta 56. – 2017. – P. 259–282.

[14] Biletsky E. Theoretical aspects of non-newtonian fluids flow simulation in food technologies / E. Biletsky, E. Petrenko, D. Semeniuk // Ukrainian Food Journal, - 2014. – 3(2). - P. 271–280.

[15] Biletsky E. Three-dimensional model of non-Newtonian fluid flow in the rectangular channel / E. Biletsky, E. Petrenko, D. Semeniuk // Ukrainian Food Journal. - 2016. – 5(3). - P. 550–560.

[16] Petrenko E. Modeling of the viscoplastic flow of a bingam fluid with transverse circulation in a rectangular channel of a worm machine / E. Petrenko E. Biletsky, D. Semeniuk // Journal of Chemistry and Technologies. – 2019. – 27(2). – P. 550–560.

[17] Кирсанов Е. А. Неньютоновское течение дисперсных, полимерных и жидкокристаллических систем: структурный подход / Е. А. Кирсанов, В. Н. Матвеенко – Москва: Техносфера, 2016. – 383с.

[18] Borzenko E. Nonisothermal filling of a planar channel with a power-law fluid / E. Borzenko, I. Ryltsev, O. Frolov, G. Shrager. // Journal of Physics: Conf. Series. - 2017. - 899. P. 1–6.

[19] Saramito P. Progress in numerical simulation of yield stress fluid flows / P. Saramito, A. Wachs // Rheologica Acta 56. – 2017. – P. 211 – 230

[20] Frank-Kamenetskii D.A. Diffusion and Heat Exchange in Chemical Kinetics / D.A. Frank-Kamenetskii - Princeton: Princeton University Press, 2015 – 384 p.

References

[1] Harlamov S. (2013) [*Gidrodinamika i teploobmen: Novyie tendentsii i perspektivyi modelirovaniya vnutrennih techeniy*]. LAP LAMBERT Academic Publishing. (in Russian).

[2] Ulyev, L. M. (2016). [*Laminar flow in coaxial conical channels*]. Kharkiv. Ukraine: NTU "KhPI". (in Ukraine).

[3] Cantelli, A. (2009). [Uniform Flow of Modified Bingham Fluids in Narrow Cross Sections]. *Journal of Hydraulic Engineering*. *135*(8). 640–650.

[4] Baptista A., Alves M.A., Coelho P.M. (2014) [Heat transfer in fully developed laminar flow of power law fluids]. *Journal of Heat Transfer*. *136*(4). 1–8.

[5] Golovanchikov, A. B., Shagarova, A. A. (2015). [Modeling the flow of a viscoplastic reaction mass in a screw reactor with a low viscosity wall layer]. *Izvestiya vuzov. Khimiya i khimicheskaya tekhnologiya*. *58*(12), 69–72.

[6] Ponomareva, M. A. Filina, M. P., Yakutonok, V. A. (2016). [Circulating flow of high viscosity non-Newtonian fluid in a single-screw extruder channel]. *Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika*., *2*(40), 97–107.

[7] Tovazhnyansky, L. L., Biletsky, E.V., Tolchinsky Yu. A. (2013) [*Modeling of flows of non-Newtonian fluids in channels of basic geometry*]. Kharkiv. Ukraine: NTU "KhPI". (in Ukraine).

[8] Kolgruber, K. (2016). [Twin-screw, co-directed extruders. Basics, technology, application] Saint Petersburg. Russian Federation: Profession. (in Russian).

[9] Ryabinin M. V., Trukhanov K. A. (2015). [The method of determining friction losses in a hydraulically smooth round pipe for pseudoplastic fluids]. *Sovremennyye problemy nauki i obrazovaniya. 1*(1). http://www.science-education.ru. (in Russian).

[10] Anandha, Rao M. (2014). *Rheology of Fluid, Semisolid, and Solid Foods: Principles and Applications*. New York, USA : Imprint: Springer.

[11] Perminov, A. V., . Lyubimova, T. P. (2014). [Stability of a stationary plane-parallel flow of a pseudoplastic fluid in a flat vertical layer]. *Vychislitel'naya mekhanika sploshnykh sred*. *7*(3). 270 – 278. (in Russian).

[12] Mitsoulis, E., Tsamopoulos, J., (2017). Numerical simulations of complex yield-stress fluid flows. *Rheologica Acta 56*. 231–258.

[13] Frigaard, I. A., Paso, K., de Souza Mendes, P., (2017) Binghams model in the oil and gas industry. *Rheologica Acta 56*. 259–282.

[14] Biletsky, E. Petrenko, E., Semeniuk, D. (2014). Theoretical aspects of non-newtonian fluids flow simulation in food technologies. *Ukrainian Food Journal*. 3(2), 271-280.

[15] Biletsky, E. Petrenko, E., Semeniuk, D. (2016). Three-dimensional model of non-Newtonian fluid flow in the rectangular channel. *Ukrainian Food Journal*. 5(3), 550-560.

[16] Petrenko, E., Biletsky, E., Semeniuk, D. (2019). Modeling of the viscoplastic flow of a bingam fluid with transverse circulation in a rectangular channel of a worm machine. *Journal of Chemistry and Technologies*. 27(2), 550-560.

[17] Kirsanov, E. A., Matveenko V. N. (2016). [*Non-Newtonian Current of Disperse, Polymer, and Liquid Crystal Systems*] Moscow. Russian Federation: Technosphere. (in Russian).

[18] Borzenko E., Ryltsev I., Frolov O. Shrager G. (2017) [Nonisothermal filling of a planar channel with a power-law fluid]. *Journal of Physics: Conf. Series*. 899. 1–6.

[19] Saramito, P., Wachs A., (2017). [Progress in numerical simulation of yield stress fluid flows]. *Rheologica Acta 56*. 211–230.

[20] Frank-Kamenetskii D.A. (2015) [*Diffusion and Heat Exchange in Chemical Kinetics*]. Princeton. USA : Princeton University Press.