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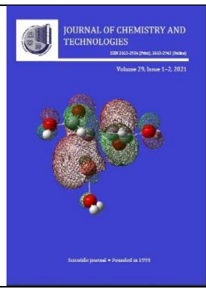

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HEAT EXCHANGE EQUATION DURING THE FLOW OF NON-NEWTONIAN LIQUIDS IN TECHNOLOGICAL EQUIPMENT CHANNELS

Eduard V. Biletsky,[1] Igor M. Ryshchenko[1], Elena V. Petrenko*,[2] Dmitrij P. Semeniuk[2]

[1] National Technical University «Kharkiv Polytechnic Institute», 2, Kyrpychova str., Kharkiv, 61002, Ukraine

[2] Kharkiv State University of Food Technology and Trade, 333 Klochkovskaya str., Kharkiv, 61051, Ukraine

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Abstract

In this article we considered the processes of heat exchange in the channels of technological equipment in the cases that are most common in machines and apparatus of the chemical and food industries. In the first case, the external environment is considered to be an infinite heat tank with a given temperature. In the second case, the role of the external environment is performed by the channel with a moving heat carrier, while the temperature of the heat carrier is not set and varies along the length of the channel. The heat transfer equation includes convective terms and terms with thermal conductivity. We formulated the heat exchange equation for the flow of non-Newtonian (viscoplastic and generalized-shifted) fluids. It is determined that: during the flow of generalized-shifted fluid in a flat channel with set wall temperatures it is necessary to define two heat transfer coefficients; in the flat channel immersed in the heat tank - four heat transfer coefficients; in the flat channel surrounded by a bypass channel - eight heat transfer coefficients. When describing heat transfer with a rectangular channel, for all these cases, it is necessary to determine respectively four, eight and sixteen heat transfer coefficients. During the flow of viscoplastic fluid, it is necessary to determine: in the flat channel with the set wall temperatures – four heat transfer coefficients; in the channel immersed in the heat tank - six heat transfer coefficients; in the channel surrounded by a bypass channel – ten heat transfer coefficients. When describing the heat transfer of a flow in a rectangular channel for the same cases, it is necessary to define respectively eight, twelve and twenty heat transfer coefficients. The heat exchange equations are a system of first-order differential equations in finite differences for the temperature of the liquid in the channel. And this is their main difference from the calculations for the cases of fixed temperatures on the walls of the straight channel and the immersion of the straight channel in the heat tank with a fixed temperature. It is shown that the fluid temperature depends on the longitudinal coordinate along the channel. In this case, the dependence of temperature on the geometric characteristics of the channel is determined by the cross-sectional area of the channel and its perimeter, as well as the ratio of geometric dimensions (width, height and length) of the channel. When performing engineering calculations, the obtained expressions allow to determine the corresponding coefficients of convective heat transfer and heat transfer during the flow of non-Newtonian fluids in the channels and with the external environment.

Keywords: heat exchange; flow; liquid; viscoplastic; generalized-shifted; channel; straight; bypass; tank.

РІВНЯННЯ ТЕПЛООБМІНУ ПРИ ТЕЧІЇ НЕНЬЮТОНІВСЬКИХ РІДИН У КАНАЛАХ ТЕХНОЛОГІЧНОГО ОБЛАДНАННЯ

Едуард В. Білецький,[1] Ігор М. Рищенко,[1] Олена В. Петренко,[2] Дмитро П. Семенюк[2]

[1] Національний технічний університет «Харківський політехнічний інститут», вул. Кирпичова, 2, Харків, 61002, Україна

[2] Харківський державний університет харчування та торгівлі вул. Клочківська, 333, Харків 61051, Україна

Антоація

Розглянуто процеси теплообміну у каналах технологічного обладнання з навколишнім середовищем у випадках, які є найбільш розповсюдженими в машинах та апаратах хімічної та харчової промисловості. У першому випадку зовнішнє середовище вважається нескінченним тепловим резервуаром із заданою температурою. У другому випадку роль зовнішнього середовища виконує канал, у якому рухається теплоносій, при цьому температура теплоносія не вважатися заданою і змінюється уздовж довжини каналу. У рівняння теплообміну входять конвективні доданки та доданки з теплопровідністю при цьому теплообмін у каналі з неньютонівською рідиною відбувається при великих значеннях числа Пекле. Рух теплоносія в каналі вважається інерційним і теж відповідає великим значенням числа Пекле. У гідродинамічному аспекті неньютонівські рідини та теплоносій рухаються в різних режимах, а в тепловому аспекті – в одному.

**Corresponding author: e-mail: elenalion71@gmail.com*

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Сформульовано рівняння теплообміну при течії неньютонівських (в'язкопластичної та узагальнено-зрушеної) рідин. Наведені рівняння теплообміну, являють собою систему диферинціальних рівнянь першого порядку в кінцевих різницях для температури рідини в каналі. І в цьому полягає їх головна відмінність від розрахунків для випадків фіксованих температ на стінках прямого каналу та занурення прямого каналу в тепловий резервуар з фіксованою температурою. Показано, що температура рідини залежить від поздовжньої координати вздовж каналу. В цьому випадку залежність температури від геометричних характеристик каналу визначається площею поперечного перетину каналу та його периметром, а також відношенням геометрічних розмірів (ширини, висоти та довжини) каналу. Отримані вирази, при проведенні інженерних розрахунків дозволяють визначати відповідні коефіцієнти тепловіддачі і теплопередачі при течії неньютоновскіх рідин в каналах і з зовнішнім середовищем.
Ключові слова: теплообмін; течія; рідина; в'язкопластична; узагальнено-зрушена; канал; прямий; обвідний; резервуар.

УРАВНЕНИЯ ТЕПЛООБМЕНА ПРИ ТЕЧЕНИЯ НЕНЬЮТОНОВСКИХ ЖИДКОСТЕЙ В КАНАЛАХ ТЕХНОЛОГИЧЕСКОГО ОБОРУДОВАНИЯ

Эдуард В. Белецкий,[1] Игорь М. Рыщенко,[2] Елена В. Петренко,[3] Дмитрий П. Семенюк[4]
[1]Национальный технический университет «Харьковский политехнический институт», ул. Кирпичева, 2, Харьков, 61002, Украина
[2]Харьковский государственный университет питания и торговли, ул. Клочковская, 333, Харьков, 61051, Украина

Аннотация
Рассмотрены процессы теплообмена в каналах технологического оборудования с окружающей средой в случаях наиболее распространенных в машинах и аппаратах химической и пищевой промышленности. В первом случае внешняя среда считается бесконечным тепловым резервуаром с заданной температурой. Во втором случае роль внешней среды выполняет канал, в котором движется теплоноситель, при этом температура теплоносителя считаться заданной и меняется вдоль длины канала. В уравнение теплообмена входят конвективные слагаемые и слагаемые с теплопроводностью при этом теплообмен в канале с неньютоновской жидкостью происходит при больших значениях числа Пекле. Движение теплоносителя в канале считается инерционным и тоже соответствует большим значением числа Пекле. В гидродинамическом аспекте неньютоновские жидкости и теплоноситель движутся в разных режимах, а в тепловом аспекте – в одном. Сформулированы уравнения теплообмена при течении неньютоновских (вязкопластичной и обобщенно-сдвиговой) жидкостей. Приведенные уравнения теплообмена, представляют собой систему диферинциальних уравнений первого порядка в конечных разностях для температуры жидкости в канале. И в этом заключается их главное отличие от расчетов для случаев фиксированных температур на стенках прямого канала и погруженонного прямого канала в тепловой резервуар с фиксированной температурой. Показано, что температура жидкости зависит от продольной координаты вдоль канала. В этом случае зависимость температуры от геометрических характеристик канала определяется площадью поперечного сечения канала и его периметром, а также отношением геометрических размеров (ширины, высоты и длины) канала. Полученные выражения, при проведении инженерных расчетов позволяют определять соответствующие коэффициенты теплоотдачи и теплопередачи при течении неньютоновских жидкостей в каналах и с внешней средой.
Ключевые слова: теплообмен; течение; жидкость; вязкопластическая; обобщенно-сдвиговая; канал; прямой; обводной; резервуар.

Introduction

The main processes of the chemical and food industries are associated with the motion of complex dispersed systems, most of which are non-Newtonian fluids. Knowledge of the structure and types of flow plays an important role in the organization of processes and allows to influence their energy efficiency by establishing rational values of hydrodynamic, thermal, mass transfer and other indicators [1]. Typically, the movement of a fluid with high viscosity occurs in the channels of the working chambers of various technological machines and depends on many parameters, namely: pressure, flow rate, shear rate, temperature, degree of mixing, dispersion, etc [2; 3]. Fluids with low viscosity values are used as intermediate heat carriers in the shells of heating devices. In all cases, materials, the viscosity of which is determined by the shear rate, differ in the great variety of their rheological properties [4].

From the technical literature analysis, we can conclude that of all the variety of non-Newtonian fluids, the most common are representatives of three classes – Bingham (viscoplastic), generalized-shifted, and power fluids [5; 6]. By the term "generalized-shifted fluids" we mean fluids, the viscosity of which depends on the shear rate in an arbitrary manner. A special case of such a liquid is a degree fluid [7]. Today there are many ways to supply and remove heat fluxes to or from the heat transfer surface of machines and devices, including the principle: tube-in-tube or with an intermediate shell. The magnitude of heat flux through a solid surface is determined by its thermal resistance and heat transfer coefficients on the sides of the media that exchange heat [8]. Such flow sections as pipes and channels were chosen by us because the pipe is the main element of heat exchangers, and the channel is the main element of the worm extruder working chamber [9; 10]. The content of this work is based on a number of results about the flow of the above-mentioned liquids in pipes and channels [9; 11–15]. In particular, [15; 16] considered the flow in the channels of flat and rectangular shapes, the boundaries of which move along themselves, as well as in the longitudinal and transverse directions. In [15–17], three-dimensional fields of flow of non-Newtonian fluids under different boundary conditions were constructed, which constitute the necessary conditions for calculating heat transfer coefficients.

Analysis of recent research and publications. Heat transfer plays an important role in the processes of the chemical and food industries. A detailed study of the structure of the heat flow allows for a high level of organization of technological processes. It is known that most of the liquids used in the production of chemical and food products have an abnormal flow, so the study of the heat transfer process of non-Newtonian fluids is very important [18–21]. Recently, the boundaries of heat transfer research have significantly expanded. In the monograph [22], the modeling of hydrodynamics and heat transfer of non-Newtonian fluids in annular channels was considered. A method for modeling the heat transfer process during extrusion of media of different rheological state was proposed. In particular, special attention was paid to the pressure flow of degree fluids in the annular channels of the cross section. The results of the temperature distribution along the cross section of the channel during the extrusion of various materials were obtained. The developed and offered mathematical model allows to carry out the optimization of technological parameters of annular profiles manufacturing.

In [23], the heat transfer and friction of a Newtonian fluid during laminar convection were considered, taking into account the change in viscosity. The influence of local heat exchange during forced and free convection in an isothermal surface cooled or heated with respect to the circulating medium was investigated. The rheological model given in [20] describes the flow curve of pseudoplastic fluids quite well. The authors found that with a change in temperature within the boundary layer in viscous liquids, the viscosity decreases sharply. In [24], numerical modeling of the nonisothermal flow of a non-Newtonian fluid between two parallel plates is presented and the influence of various actions on the development of the thermal boundary layer is analyzed. In [25], a theoretical study of the hydrodynamics and heat exchange of non-Newtonian fluids near the cooling isothermal surface under free laminar convection, taking into account the change in fluid viscosity with temperature. A power rheological model of the liquid was used in the study. Some works are devoted to the study of structural and mechanical characteristics in the displacement of non-Newtonian materials in the radial channels of screw machines. Thus, in [26], the nonisothermal flow of non-Newtonian fluid in an auger machine was considered, taking into account the radial gaps between the auger and the machine body. The study determined the behavior of the material in the process of movement in the channels. The obtained formulas allow to determine the temperature of the liquid, the flow rate, the pressure drop, as well as to determine the required power of the machine. But the above formulas need to be clarified, depending on the given rheological state of the liquid.

The authors in [27] considered the Powell-Eyring rheological model of flow in a curved channel, studied the physical phenomena that occur when the temperature changes locally, and performed numerical calculations of the dependence of viscosity on temperature. The analysis of works on the study of heat transfer processes of non-Newtonian materials allows us to draw the following conclusions. There is no clear algorithm that would allow to perform calculations that have been confirmed experimentally. The main macrodynamic and macrokinetic characteristics of the flow are subject to detailed study of the following: flow profile and velocity distribution, their gradients, dependence on shear on shear stress, type of channel or pipe geometry, dissipation value, changes in rheological properties from pressure, temperature. The lack of validated engineering and calculation methods for determining the rheodynamic and heat exchange parameters of non-Newtonian fluids in channels of complex geometry for various cases of heat exchange with the environment significantly hinders their further wide use as heat carriers in the chemical and food industries. The obtained data will allow to better organize the technological process in order to increase the efficiency of heat transfer and manage its parameters, to use in the design of machinery and apparatus of the chemical and

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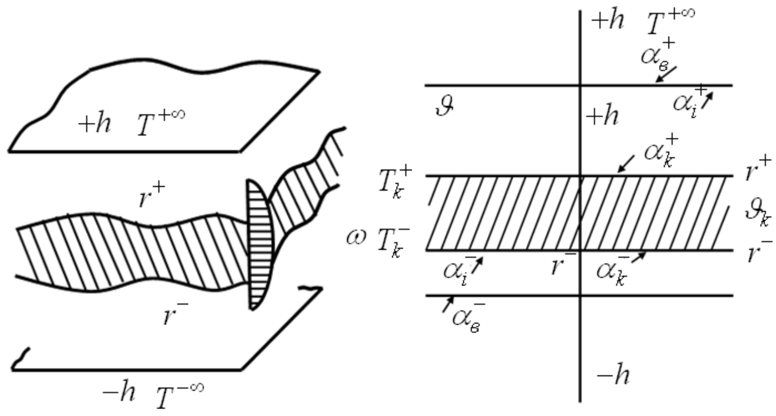
food industries, to influence the energy efficiency and material consumption of equipment.

Results of the research and their discussion

In this paper, we consider the processes of heat exchange in the channels of technological equipment with the environment in cases that are most common in machines and apparatus of the chemical and food industries [1]. In the first case, the external environment is considered to be an infinite heat reservoir with a given temperature. In the second case, the role of the external environment is performed by the channel in which the heat carrier moves, while the temperature of the heat carrier is not considered set and varies along the length of the channel. The heat transfer equation includes convective terms and terms with thermal conductivity, while heat exchange in the channel with a non-Newtonian fluid occurs at large values of the Hell number [21], despite the fact that both viscoplastic and generalized-shifted fluids have high viscosity [28]. The movement of the heat carrier in the channel is considered inertial and also corresponds to the large values of the Hell number [21]. Thus, in the hydrodynamic aspect, non-Newtonian fluids and heat carrier move in different modes, and in the thermal aspect - in one [1; 7; 22].

The three-dimensional and schematic view of the heat exchange of the viscoplastic fluid flow in the channel immersed in the heat reservoir is shown in Fig. 1.



a b

Fig. 1. Graphic interpretation of the viscoplastic fluid flow heat exchange in the channel immersed in the heat tank: a - three-dimensional view; b - diagram of the flow and characteristics of heat exchange with the heat tank

Graphic interpretation of the heat transfer of the generalized-shifted fluid in the channel, which is immersed in the heat tank, is shown in Fig. 2.

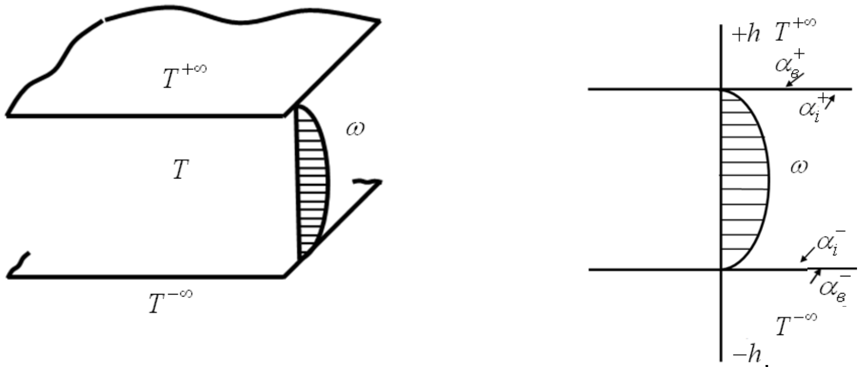


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Fig. 2. Graphic interpretation of the generalized-shifted fluid heat transfer in the channel immersed in the heat tank: a - three-dimensional view; b - diagram of the flow and characteristics of heat exchange with the heat tank

Graphical interpretation for the case of heat transfer between non-Newtonian fluid in the channel of process equipment and the heat carrier in another channel of this equipment is presented in Fig. 3–5.

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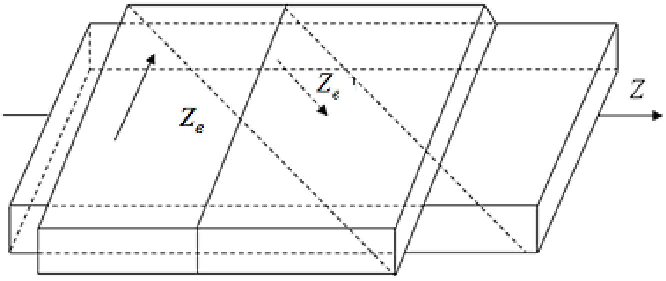
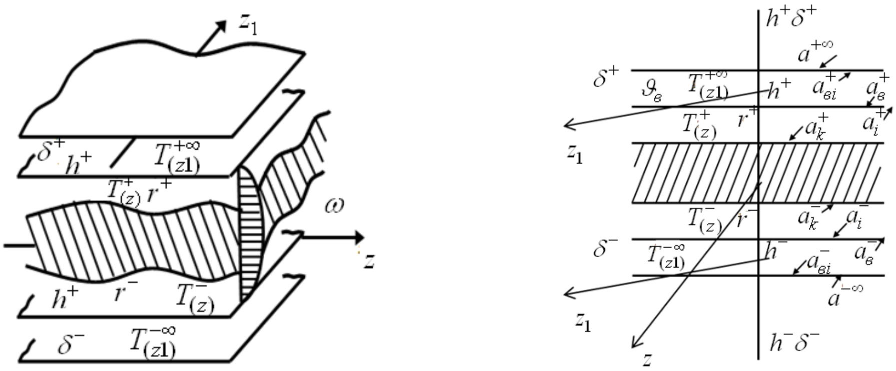


Fig. 3. Schematic representation of the straight channel and bypass channel relative position

We consider the general case in which the channel where the heat carrier moves is at an appropriate angle relative to the channel with non-Newtonian fluid (see Fig. 3). Given this circumstance, two different longitudinal coordinates z and z_e, which are interdependent, are introduced in the description. The longitudinal coordinate z is the coordinate that is read along the axis of the channel with non-Newtonian fluid, and the coordinate z_e will be assigned to the bypass channel, with the heat carrier. If the angle of rise of the enveloping channel is zero, it means that both channels interact through the direct flow or countercurrent flow. If the angle of the enveloping channel rise is equal to $\pi/2$, then both channels interact by the method of cross-flow [16].

The distribution of heat transfer coefficients during the flow of viscoplastic fluid in the straight and bypass channels is shown in Fig. 4.



a b

Fig. 4. Distribution of heat transfer coefficients during the viscoplastic fluid flow: a – three-dimensional view of straight and bypass channels; b – diagram of the flow and characteristics of heat transfer with the bypass channel

The heat transfer coefficients distribution during the flow of generalized-shifted fluid in the straight and bypass channels is shown in Fig. 5.

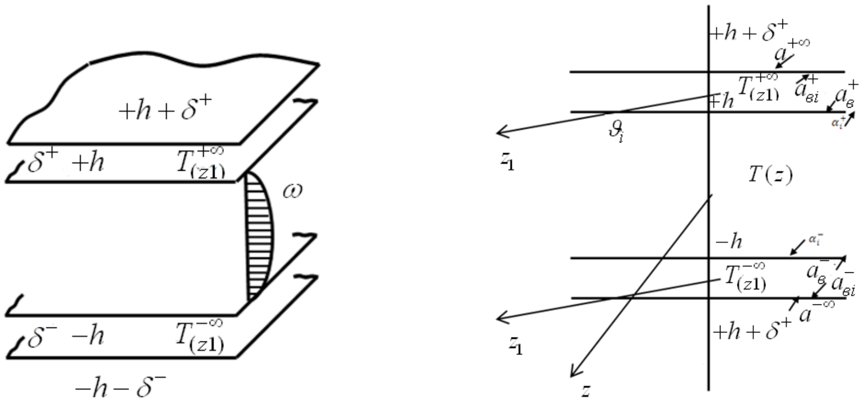


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Fig. 5. Distribution of heat transfer coefficients during the flow of generalized-shifted fluid: a – three-dimensional view of straight and bypass channels; b – diagram of the flow and characteristics of heat transfer with the bypass channel

When considering heat transfer with a viscoplastic liquid, six heat transfer coefficients for the heat reservoir and ten heat transfer coefficients for the bypass channel with the heat carrier should be used (see Figures 1b and 4b). It should be noted that during the heat exchange in the core of a viscoplastic fluid, the heat is transferred by the mechanism of thermal conductivity [1].

When describing the heat exchange processes with generalized-shifted fluid, four heat transfer coefficients should be used (see Fig. 2b) for the heat tank, and eight heat transfer coefficients for the bypass channel with the heat carrier (see Fig. 5b).

For heat exchange of viscoplastic fluid in a channel with a wall with a given temperature, the following equations are true:

$$\dot{V}^+ \rho c_p \frac{dT^+}{dz} = \dot{e}^+ S^+ + \alpha_i^+ \left(T_\Gamma^+ - T^+\right) \Pi^+ + \alpha_k^+ \left(T^+ - T_k^+\right) \Pi_k^+;$$
$$\dot{V}^- \rho\, c_p \frac{dT^-}{dz} = \dot{e}^- S^- + \alpha_i^- (T_\Gamma^- - T^-) \Pi^- + \alpha_k^- (T^- - T_k^-) \Pi_k^- \qquad (1)$$
$$\frac{\partial^2 T_k^\pm}{\partial y^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \left.\frac{\partial T_k^\pm}{\partial y}\right|_{\Gamma^\pm} = \alpha_k^\pm (T_\Gamma - T^\pm)\Big|_{\Gamma^\pm},$$

where $\dot{V}^\pm$ – the flow rate of the fluid part above and below the flow core, m³/s; $T^\pm$ – the temperature of the liquid part above and below the flow core, °C; $\dot{e}^\pm$ – specific energy dissipations of the fluid part above and below the flow core J/m³; $S^\pm$ – the cross-sectional areas of the fluid part above and below the flow core, m²; $T_k^\pm$ – core temperatures adjacent to the upper and lower parts of the liquid part, °C; $T_\Gamma^\pm$ – temperatures of the upper and lower walls of the channel, °C, where $"\Gamma" = hy,\ ax$; $\alpha_i^\pm$ – heat transfer coefficients on the walls on the lines y = ± h, J/m² °C; $\alpha_k^\pm$ – heat transfer coefficients at the boundaries of the flow core on the lines y = Γ±, J/m² °C; $\Pi^\pm$ – the length of the perimeters of the channel walls, m; $\Pi_k^\pm$ – the length of the perimeters of the intersection boundaries of the flow core, m; λ_k – thermal conductivity coefficients of the liquid part and the core of the viscoplastic fluid, J/m °C.

If the temperatures of the channel walls are known (which happens most often), the heat exchange equations take the following form:

$$\dot{V}^+ \rho\, c_p \frac{dT^+}{dz} = \dot{e}^+ S^+ + K^+ (T^{+\infty} - T^+) \Pi^+ + K_k^+ (T^+ - T_k^+) \Pi_k^+,$$
$$\dot{V}^- \rho\, c_p \frac{dT^-}{dz} = \dot{e}^- S^- + K^- (T^{-\infty} - T^-) \Pi^- + K_k^- (T^- - T_k^-) \Pi_k^-, \qquad (2)$$
$$\frac{\partial^2 T_k^\pm}{\partial y^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \left.\frac{\partial T_k^\pm}{\partial y}\right|_{\Gamma^\pm} = \alpha_k^\pm (T_\Gamma - T^\pm)\Big|_{\Gamma^\pm},$$

where $T^{\pm\infty}$ – the temperature of the heat tank above and below the channel, °C; $K^\pm$ – heat transfer coefficients between the parts of the heat tank and the liquid part of the viscoplastic fluid above and below the flow core, J/m² °C; $K_k^\pm$ – heat transfer coefficients between the liquid part and the core of the viscoplastic fluid, J/m² °C.

Heat transfer coefficients are determined from heat transfer coefficients according to the usual rules that are known from the literature [1, 7, 18]. The equation of heat exchange in the channel of the generalized-shifted fluid with the set temperatures of walls has yhe following form:

$$\dot{V} \rho\, c_p \frac{dT}{dz} = \dot{e} S + \alpha_i^+ (T_h^+ - T) \Pi^+ + \alpha_i^- (T_h^- - T) \Pi^-, \qquad (3)$$

in which the meaning of all notations is the same as in (1) and (2), only there is no difference between the upper and lower flows relative to the core of the flow.

The equation of heat transfer in the channel with the heat tank can be represented as follows:

$$\dot{V} \rho\, c_p \frac{dT}{dz} = \dot{e} S + K^+ (T^{+\infty} - T) \Pi^+ + K^- (T^{-\infty} - T) \Pi^- \qquad (4)$$

The recording of the heat transfer equations with the given wall temperatures for the flow of viscoplastic fluid in a rectangular channel must take into account the cross section of the channel. The number of heat transfer coefficients is doubled.

The corresponding partition of the flow of the generalized-shifted fluid was considered in the paper [29]. In the case of longitudinal-transverse flow, the partitioning of the longitudinal part of the flow and the partitioning of the transverse part of the flow may not coincide.

The heat exchange equations of the viscoplastic fluid flow in a rectangular channel with given wall temperatures have the following form:

$$\begin{aligned}
&\dot{V}_y^+ \rho c_p \frac{dT_y^+}{dz} = \dot{e}_y^+ S_y^+ + \alpha_{iy}^+ (T_{hy}^+ - T_y^+) \Pi_y^+ + \alpha_{ky}^+ (T_y^+ - T_{k\Gamma}^+) \Pi_{ky}^+ ,\\
&\dot{V}_y^- \rho c_p \frac{dT_y^-}{dz} = \dot{e}_y^- S_y^- + \alpha_{iy}^- (T_{hy}^- - T_y^-) \Pi_y^- + \alpha_{ky}^- (T_y^- - T_{k\Gamma}^-) \Pi_{ky}^- ,\\
&\dot{V}_x^+ \rho c_p \frac{dT_x^+}{dz} = \dot{e}_x^+ S_x^+ + \alpha_{ix}^+ (T_{ax}^+ - T_x^+) \Pi_x^+ + \alpha_{kx}^+ (T_x^+ - T_{k\Gamma}^+) \Pi_{kx}^+ ,\\
&\dot{V}_x^- \rho c_p \frac{dT_x^-}{dz} = \dot{e}_x^- S_x^- + \alpha_{ix}^- (T_{ax}^- - T_x^-) \Pi_x^- + \alpha_{kx}^- (T_x^- - T_{k\Gamma}^-) \Pi_{kx}^- ,\\
&\frac{\partial^2 T_k^\pm}{\partial x^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \frac{\partial T_k^\pm}{\partial y}\bigg|_{\Gamma_y^\pm} = \alpha_{ky}^\pm (T_{ky}^\pm - T_y^\pm)\Big|_{\Gamma_y^\pm} ,\\
&\frac{\partial^2 T_k^\pm}{\partial y^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \frac{\partial T_k^\pm}{\partial x}\bigg|_{\Gamma_x^\pm} = \alpha_{kx}^\pm (T_{kx}^\pm - T_x^\pm)\Big|_{\Gamma_x^\pm} .
\end{aligned} \tag{5}$$

The heat exchange equation of the viscoplastic fluid flow in the heat tank with the set temperatures is as follows:

$$\begin{aligned}
&\dot{V}_y^+ \rho c_p \frac{dT_y^+}{dz} = \dot{e}_y^+ S_y^+ + K_y^+ (T_y^{+\infty} - T_y^+) \Pi_y^+ + K_{ky}^+ (T_y^+ - T_{ky}^+) \Pi_{ky}^+ ,\\
&\dot{V}_y^- \rho c_p \frac{dT_y^-}{dz} = \dot{e}_y^- S_y^- + K_y^- (T_y^{-\infty} - T_y^-) \Pi_y^- + K_{ky}^- (T_y^- - T_{ky}^-) \Pi_{ky}^- ,\\
&\dot{V}_x^+ \rho c_p \frac{dT_x^+}{dz} = \dot{e}_x^+ S_x^+ + K_x^+ (T_x^{+\infty} - T_x^+) \Pi_x^+ + K_{kx}^+ (T_x^+ - T_{kx}^+) \Pi_{kx}^+ ,\\
&\dot{V}_x^- \rho c_p \frac{dT_x^-}{dz} = \dot{e}_x^- S_x^- + K_x^- (T_x^{-\infty} - T_x^-) \Pi_x^- + K_{kx}^- (T_x^- - T_{kx}^-) \Pi_{kx}^- ,\\
&\frac{\partial^2 T_k^\pm}{\partial x^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \frac{\partial T_k^\pm}{\partial y}\bigg|_{\Gamma_y^\pm} = \alpha_{ky}^\pm (T_{ky}^\pm - T_y^\pm)\Big|_{\Gamma_y^\pm} ,\\
&\frac{\partial^2 T_k^\pm}{\partial y^2} + \frac{\partial^2 T_k^\pm}{\partial z^2} = 0; \qquad -\lambda_k \frac{\partial T_k^\pm}{\partial x}\bigg|_{\Gamma_x^\pm} = \alpha_{kx}^\pm (T_{kx}^\pm - T_x^\pm)\Big|_{\Gamma_x^\pm} .
\end{aligned} \tag{6}$$

The heat transfer of the generalized-shifted fluid in a rectangular channel with given wall temperatures can be represented by the following equations:

$$\begin{aligned}
&\dot{V}_y^+ \rho c_p \frac{dT_y^+}{dz} = \dot{e}_y^+ S_y^+ + \alpha_{iy}^+ (T_h^+ - T_y^+) \Pi_y^+ ,\\
&\dot{V}_y^- \rho c_p \frac{dT_y^-}{dz} = \dot{e}_y^- S_y^- + \alpha_{iy}^- (T_h^- - T_y^-) \Pi_y^- ,\\
&\dot{V}_x^+ \rho c_p \frac{dT_x^+}{dz} = \dot{e}_x^+ S_x^+ + \alpha_{ix}^+ (T_a^+ - T_x^+) \Pi_x^+ ,\\
&\dot{V}_x^- \rho c_p \frac{dT_x^-}{dz} = \dot{e}_x^- S_x^- + \alpha_{ix}^- (T_a^- - T_x^-) \Pi_x^- .
\end{aligned} \tag{7}$$

The heat exchange equation (7) of the generalized-shifted fluid in the case of flow in the heat tank takes the following form:

$$\begin{aligned}
&\dot{V}_y^+ \rho c_p \frac{dT_y^+}{dz} = \dot{e}_y^+ S_y^+ + K_y^+ (T_y^{+\infty} - T_y^+) \Pi_y^+ ,\\
&\dot{V}_y^- \rho c_p \frac{dT_y^-}{dz} = \dot{e}_y^- S_y^- + K_y^- (T_y^{-\infty} - T_y^-) \Pi_y^- ,\\
&\dot{V}_x^+ \rho c_p \frac{dT_x^+}{dz} = \dot{e}_x^+ S_x^+ + K_x^+ (T_x^{+\infty} - T_x^+) \Pi_x^+ ,
\end{aligned} \tag{8}$$

$$\dot{V}_x^- \rho\, c_p \frac{dT_x^-}{dz} = \dot{e}_x^- S_x^- + K_x^- (T_x^{-\infty} - T_x^-) \Pi_x^- .$$

Equations (2), (4), (6), (8) in the case where the flat or rectangular channels are covered by a bypass channel in which the heat carrier flows, should be supplemented with equations describing changes in the temperature of the heat tank. For the sake of brevity, only the heat transfer equation for the heat tank is considered. These equations look like this:

$$\dot{V}_e^+ \rho_e\, c_{pe} \frac{dT_e^+}{dz_e} = K^+ (T_e^+ - T^+) \Pi^+ + K_e^+ (T_e^+ - T_{ee}^+) \Pi_{ee}^+ ,$$
$$\dot{V}_e^- \rho_e\, c_{pe} \frac{dT_e^-}{dz_e} = K^- (T_e^- - T^-) \Pi^- + K_e^- (T_e^- - T_{ee}^-) \Pi_{ee}^- . \qquad (9)$$

where $T_e^{\pm}$ – the temperature of the channel with the heat carrier, °C; c_{pe} – heat capacity of the heat carrier, J/kg °C; «*ee*» – index, which means the external environment in accordance to the channel with the heat carrier.

The temperatures $T_e^{\pm}$ in the case of flow in a heat tank with fixed temperatures take the values $T^{\pm\infty}$. For a rectangular channel, the number of equations of type (9) is doubled; as a result, all values are supplied with indices *x* and *y*. The same must be done for the flow of generalized-displaced fluid, and the relationship between the variables z and z_e will be discussed below.

If the channel with a very viscous liquid is surrounded by a bypass channel, then for both viscoplastic and generalized-shifted liquids, a single temperature $T_e(z_e)$ will be used instead of individual temperatures $T_e^{\pm}$, but which is taken at different points in the z_e coordinate.

The form of equations (1) - (9) shows that the longitudinal temperature field can be determined by calculating the values of specific dissipations. For this purpose it is necessary to construct border lines of division of a rectangle in section of the channel into corresponding regions and to calculate their areas; calculate the integrals of the specific dissipation by regions; calculate heat transfer coefficients on solid walls and boundaries (for viscoplastic fluid), calculate heat transfer coefficients; solve the problem of thermal conductivity in the core of a viscoplastic fluid. But first, it is necessary to obtain the heat exchange equation for an arbitrarily oriented direction of heat transfer relative to the main channel. Such conditions are realized at heat exchange in the straight and bypass channels with an arbitrary angle of rise of a helical line of its axis.

Longitudinal coordinates z and z_e do not coincide in the bypass channel and the main channel. The relationship between the longitudinal coordinates z and z_e is established through the angle of rise φ and has the following form:

$$z_e = z\sqrt{1 + tg^2\varphi} . \qquad (10)$$

If we select a certain section of the straight channel, then the different sides of this section will correspond to different cross sections of the bypass channel, which are located one after the other and fill some space in the coordinate z_e. For each set of sections of the bypass channel, it is possible to specify the average cross-section, which corresponds to a certain coordinate z_e. Repeating this procedure for all sides of the cross section of the main channel, we can conclude that the rectangular section of the straight channel is surrounded by a section of the channel with temperature T_e, which should be taken at four different points in the z_e. coordinate.

The z_{ei} arguments for the temperature T_e take the following values:

$$z_{e1} = \xi_\varphi z ; \qquad z_{e2} = \xi_\varphi z + \left(\frac{a+h}{2} + \delta_e\right)\sqrt{1-\xi_\varphi^2} ;$$
$$z_{e3} = \xi_\varphi z + \left(\frac{2a+2h}{2} + 2\delta_e\right)\sqrt{1-\xi_\varphi^{\,2}} ; \qquad (11)$$
$$z_{e4} = \xi_\varphi z + \left(\frac{3a+3h}{2} + 3\delta_e\right)\sqrt{1-\xi_\varphi^2} ; \qquad \xi_\varphi \equiv 1 + tg^2\phi$$

where a, h, δ – dimensions of the rectangle of the straight channel and the thickness of the wrapping channel, m.

As an example of heat exchange equations, we can consider the case of heat exchange between a generalized-shifted fluid and a heat carrier in a flat channel. This simplest case to record demonstrates all the basic features of cross-exchange. In this case, the equations have a rather complex form:

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- for the internal channel:

$$\dot{V}\rho\, c_p \frac{dT}{dz} = \dot{e}S + K_1(T_1^\infty - T)a + K_2(T_2^\infty - T)h + K_3(T_3^\infty - T)a + K_4(T_4^\infty - T)h\,,$$

- for the channel with the heat carrier:

$$\dot{V}_e \rho_e\, c_{pe} \frac{dT_y^+}{dz} = -K_1(T_1^\infty - T)a - K_2(T_2^\infty - T)h - K_3(T_3^\infty - T)a - K_4(T_4^\infty - T)h - K_{e1}(T_1^\infty - T_{1ee})(a + 2\delta_e) -$$

$$-K_{e2}(T_2^\infty - T_{2ee})(h + 2\delta_e) - K_{e3}(T_3^\infty - T_{3ee})(a + 2\delta_e) - -K_{4e}(T_4^\infty - T_{4ee})(h + 2\delta_e) \qquad (12)$$

$$K_i \equiv K_i(z_{ei})\,; \qquad K_{ie} = K_{ie}(z_{ei})\,, \qquad T_i^\infty = T_i^\infty(z_{ei})\,,$$

where T_{iee} – the value of the environment temperature that surrounds the bypass channel, °C.

The second equation in (12) retains its form for describing flows of both generalized-shifted and viscoplastic fluids in a straight channel. If we add to it the equations for the straight channel and replace its terms with heat transfer coefficients by the sums of the corresponding terms which are taken at different points in the z_{ei}. Coordinate, then we obtain equations that are similar to equations (1)–(9).

Conclusion

The proposed calculation technique allows the calculation of heat exchange parameters in complex three-dimensional flows in channels with arbitrary velocity distribution for the main cases of heat exchange: a channel immersed in a heat tank with a given temperature; a channel surrounded by a bypass channel with a heat carrier. The heat exchange equations are a system of first-order differential equations in finite differences for the temperature of the liquid in the channel. And this is their main difference from the calculations for the cases of fixed temperatures on the walls of the straight channel and the immersion of the straight channel in the heat tank with a fixed temperature. In addition, it is shown that the temperature of the liquid depends on the longitudinal coordinate along the channel. In this case, the dependence of temperature on the geometric characteristics of the channel is determined by the cross-sectional area of the channel and its perimeter, as well as the ratio of the geometric dimensions of the channel (width, height and length).

The following formulations allow the calculation of thermal and hydrodynamic characteristics (convective heat transfer and heat transfer coefficients, Nusselt numbers, dissipative heat dissipation, solid core temperatures, flow velocities) during the flow of viscoplastic, generalized-shifted fluids channels with different geometry in chemical and food equipment.

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