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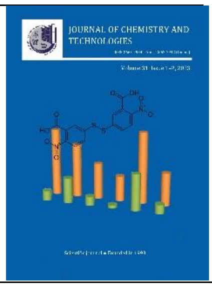

UDC 541.1

CHAOS CONTROL VIA EXTERNAL PERIODIC FORCING IN AN AUTOCATALYTIC DISSIPATIVE CHEMICAL SYSTEM

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Abstract

In this paper, we study the controlling of chaotic behaviours in an autocatalytic dissipative chemical system governed by a forced modified Duffing – Van der Pol (DVP) oscillator driven by various sinusoidal periodic forces. The external sinusoidal periodic forces considered are sine wave, modulus of sine wave and rectified sine wave. The effects of the sinusoidal forces and the perturbation parameter Γ on chaotic motions of the chemical system have been strongly analyzed. Controlling of chaotic behaviours have been investigated through bifurcation structures, Lyapunov exponent, phase portrait, Poincar´e section and time series. Coexistence of several attractors and hysteresis phenomenon have been studied in detail in the system with sinusoidal excitations.

Keywords: Autocatalytic dissipative chemical system; bifurcation; chaos; hysteresis; sinusoidal excitation.

КЕРУВАННЯ ХАОСОМ ЗА ДОПОМОГОЮ ЗОВНІШНЬОГО ПЕРІОДИЧНОГО ПРИМУСУ В АВТОКАТАЛІТИЧНІЙ ДИСИПАТИВНІЙ ХІМІЧНІЙ СИСТЕМІ

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Анотація

У цій статті вивчено керування хаотичною поведінкою в автокаталітичній дисипативній хімічній системі, керованій примусово модифікованим осцилятором Даффінга – Ван дер Поля (DVP), що приводиться в дію різними синусоїдальними періодичними силами. Розглянуто зовнішні синусоїдальні періодичні сили: синусоїдальна хвиля, модуль синусоїди та випрямлена синусоїдальна хвиля. Ретельно проаналізовано вплив синусоїдальних сил і параметра збурення Γ на хаотичні рухи хімічної системи. Керування хаотичною поведінкою досліджено за допомогою біфуркаційних структур, експоненти Ляпунова, фазового портрета, перерізу Пуанкаре та часових рядів. Детально вивчено співіснування кількох атракторів та явище гістерезису в системі з синусоїдальними збудженнями.

Ключові слова: автокаталітична дисипативна хімічна система; біфуркація; хаос; гістерезис; синусоїдальне збудження.

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Introduction:

A chemical reaction is called autocatalytic if at least one of the reaction products acts as a catalyst in the same or in one of the coupled reactions. Reactions of this type have the property that the rate equations are nonlinear, that is, the reaction is very slow in the beginning but steadily increases as more products are formed. The simplest autocatalytic chemical reaction is of the following form:

$$A + p\,B \rightarrow (p+1)B \quad (1)$$

where A is the reactant and B is the autocatalyst, and the integer $p \geq 1$ is the order of the reaction (number of autocatalyst molecules involved in a reaction). While the most common case is $p = 1$, the higher order reactions with $p \geq 2$ have been considered in recent years [1; 2]. There are many chemical and biological reactions, which are autocatalytic. Some examples are, (i) the reaction of oxalic acid with permanganate, here Mn^{2+} ions are products and also act as a catalyst (ii) haloform reaction (iii) decomposition of Arsine is catalysed by Arsenic, which is produced in the reaction $2As\ H_3 \rightarrow 2As + 3H_2$, (iv) binding of oxygen by haemoglobin, (v) DNA replication (vi) Formoss or Butlerov reaction, (vii) Tin pest (viii) Vinegar syndrome and (ix) photographic processing of silver halide film/paper [3–7].

Chemical systems may exhibit chaotic behaviour if they contain certain elements of dynamical feedback. While chaos is intriguing, it is not clear what role it plays in real chemical processes, in living systems and otherwise. One suggestion is that chaotic systems possess a virtually unlimited wealth of dynamical behaviours and these behaviours can be brought under control in a deliberate and selective manner. Controlling chaos can be understood as a process or mechanism which enhances existing chaos or creates chaos in a dynamical system when it is useful or beneficial and suppresses it when is harmful. Controlling chaos is very important in many circumstances from the point of view of preventing disaster and collapse in a dynamical system. In recent years, after the pioneering work of Ott, Grebogi and Yorke (OGY) [8], controlling chaos has become more and more interesting in academic research and practical applications [9–16]. The first control experiment in chemical chaos was carried out in a BZ reaction by the group of Showalter in 1993. The authors applied a map-based, proportional-feedback algorithm to stabilize periodic behaviour in the chaotic regime of an oscillating BZ reaction [17]. The dynamical behaviours of the Brusselator chemical system with impulse input [18], amplitude and frequency modulated forces [7; 19] and with different shape of periodic forces [15] have been also investigated. Recently, Suddalai Kannan et al. [6] studied the control of chaos and bifurcation by nonfeedback methods in an autocatalytic chemical system.

To derive a chaotic system trajectory to a periodic orbit, one may tune the system key parameters and monitor and control the resulting dynamics. However, as common practice in chaotic system, a simple way is to use external periodic forcing to dominate and modify the undesired system dynamics. It is conceivable that a large enough periodic signal would render a chaotic system periodic. But the signals of small amplitude are usually preferable. Reducing chaos in a dynamical system by applying either an external weak periodic signal or a random signal has proven not only possible but quite effective for many control purposes [14; 15; 20–24]. The present paper is organized as follows. In Section 2, we present the chemical model and its kinetic equation. In Section 3, we present types of sinusoidal periodic forces and their associated mathematical representations. In Section 4, we analyze the control of chaos caused by various sinusoidal forces. In Section 5, we analyze the phenomenon of hysteresis due to various sinusoidal periodic forces. Finally, Section 6 contains our conclusion.

Chemical model and its equation of motion

Nonlinear dynamics has become increasingly important in chemical kinetics. A variety of examples are known to exhibit periodic and chaotic variations in the concentrations of reacting species. It is well known that oscillatory and chaotic behaviours are associated with onlinear phenomena and the corresponding mathematical models are governed by deterministic differential equations. The differential equation models for chemical schemes have been separated traditionally from those for physical systems. There have been many methods devoted to set up the relationship between chemical system and physical system by means of transformation. Of these methods, Samardzijia's nonlinear transformation is one of the mostly used methods and has been extensively quoted [25]. This methods succeeds in converting some famous models such as Van der Pol-Duffing [26], Lorenz [27], Rossler spiral chaos [28], forced negative stiffness Duffing [29], a Chua's circuit [30] etc. in to mass action chemical schemes which preserve

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the phase space qualitative features of the original system. The generic model for nonlinear chemical oscillations used in the study based on the kinetic scheme which can be described by the following chain of equations:

$$A \overset{K_1}{\to} U \quad (2)$$
$$B + U \overset{K_2}{\to} 2U \quad (3)$$
$$D + U \overset{K_3}{\to} Products \quad (4)$$
$$U \overset{K_4}{\to} U' \quad (5)$$
$$B + U' \overset{K_5}{\to} V \quad (6)$$
$$V \overset{K_6}{\to} U' + Products \quad (7)$$

In a flow reactor, the incoming fluxes of the respective species *A*, *B* and *D* and the inverse of the resident time, K_i, $i = 1, 2…6$, are controlled externally. It has been shown that, if one derives the kinetic equations under the assumptions of the law of mass action, that steps (1–4) may give a bistability and that steps (4–6) may be handled as a feedback on the constant parameter of the autocatalytic step. Based upon the laws of mass action and conservation and assuming that the sink of the product is a first order reaction, the simple mathematical model for self-oscillations in some nonlinear chemical system with external periodic signal defined as follows:

$$\ddot{x} + \mu\,(1 - x^2)\,\dot{x} + \alpha\,x + \beta\,x^3 + \Gamma = F(t) \quad (8)$$

where x is proportional to the concentration of species U and represents the displacement, $\dot{x}$ and $\ddot{x}$ are the velocity and acceleration respectively. Parameters μ, α, β and Γ respectively denote the damping coefficient, linear and cubic nonlinear restoring parameters and nonlinear parameter. $F(t)$ is an external sinusoidal periodic forces. Equation (8) is a forced modified Duffing-vander Pol (DVP) oscillator equation. Recently, many authors investigated certain nonlinear phenomena and the active control of chaotic oscillations in this nonlinear chemical system modeled by a modified Van der Pol-Duffing oscillator [12; 31]. For the particular case where the nonlinear term $\Gamma = 0$ and Eq. (8) is reduced to the classical Duffing-vander Pol oscillator equation. The nonlinear parameter Γ mark the difference between the oscillator Eq. (8) and the equation of classical Van der Pol – Duffing oscillator. This classical driven DVP oscillator has been widely studied in the context of various physical, chemical and engineering problems [32–37].

Types of sinusoidal periodic forces. The mathematical representations of the sinusoidal periodic forces are the following:

(a) Sine wave

The sine wave is represented by

$$F(t) = F\,(t + 2\pi/\omega) = f\sin\omega t.$$

(b) Modulus of sine wave

The modulus of sine wave is given as

$$F(t) = f|\sin(\omega t/2)|.$$

(c) Rectified sine wave

The mathematical representation of rectified sine wave is

$$F(t) = \begin{cases} f, & 0 < t < \pi/\omega \\ 0, & \pi/\omega < t < 2\pi/\omega, \end{cases} \quad (11)$$

where f and ω are the amplitude and frequency of the sinusoidal periodic force

Controlling of chaos due to various sinusoidal periodic forces.

(i) *Chaos control in the system without Γ (ie. $\Gamma = 0$).* First we analyze the controlling of chaos due to the various sinusoidal periodic forces applied on the system Eq. (8) with $\Gamma = 0$. When $\Gamma = 0$, the system (Eq.8) is reduced to classical DVP oscillator. For our numerical calculation, we fix the parameters values as $\alpha = -1.0$, $\beta = 5.0$, $\mu = 0.4$ and $\omega = 1.0$. Equation (8) is solved with different sinusoidal forces by fourth order Runge-Kutta method with time step $(2\pi/\omega)/200$. Numerical solution corresponding to first 500 drive cycle is left as transient. We analyzed the behaviors of the system by varying the forcing amplitude of the each periodic sinusoidal force. Figure 1 shows leading Lyapunov exponent (λ_m) diagrams of various sinusoidal forces. The maximal Lyapunov exponent (λ_m) is computed using the algorithm given in ref [38]. If $\lambda_{max} < 0$, the disturbed trajectory will eventually be attracted to a stable periodic orbit. $\lambda_{max} > 0$ reveals an unstable and chaotic trajectory and $\lambda_{max} = 0$ means that the disturbed oscillation and the original oscillation stay apart be a constant mean distance for an indefinite duration until perturbed again. The observed dynamical states over the range $f \in [0.5]$ are listed in table 1.

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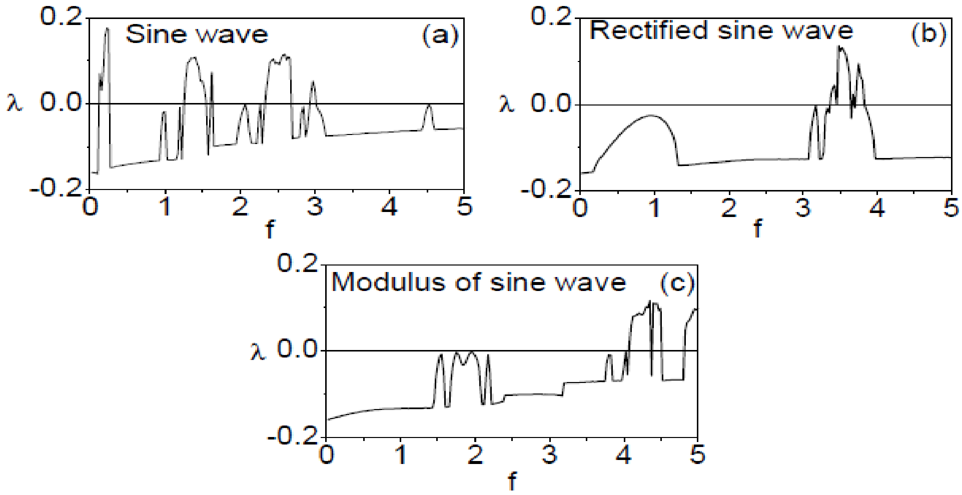

Fig. 1. Lyapunov exponents λ versus the parameter f for the system (Eq.8) driven by a (a) sine wave (b) rectified sine and (c) modulus of sine wave forces. The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4,Γ = 0 and ω = 1.0.

When the system (Eq.8) driven by a sine wave force, the system has a leading Lyapunov exponent $\lambda \approx 0.198$ at $f = 0.01$. Now the sine wave force is replaced by a rectified sine wave and modulus of sine wave forces in the system (Eq.8), the leading exponent is shown in Figs.1(b) and 1(c). It can be observed from the figures that a significant reduction in λ is obtained. Significant suppression of chaos is achieved when the rectified sine wave and modulus of sine wave forces are turned on, which is clearly seen in Figs.1(b) and 1(c).

Table 1

The dynamics of the system (Eq.8) for the different ranges of *f* with ω = 1

Types of forces	Range of f	Sign of λ_m	Dynamics
sine wave	(0.25-1.25);(1.71-2.5);(2.75-2.90); (3.1-5)	0 - - -	periodic orbits
	(0-0.25);(1.25-1.71);(2.5-2.9)	+ + + +	chaotic orbits
Rectified sine wave	(0-3.25);(3.81-5)	- -	periodic orbits
	(3.25-3.81)	+	chaotic orbit
Modulus of sine wave	(0-4.0);(4.5,4.75)	- 0 -	periodic orbits
	(4.0-4.5);(4.75-5)	+ +	chaotic orbits

(ii) *Chaos control in the system with Γ (ie. Γ ≠ 0).* In this section, we analyze the conditions for suppressing chaotic oscillations or instabilities in the modified and forced DVP oscillator (ie. in Eq.(8), $\Gamma \neq 0$) driven by various sinusoidal periodic forces, by doing the numerical simulation of the system (Eq.8).

(a) Effect of sine wave force

First we consider the effect of the force f sinωt. Figure 2 shows the maximal Lyapunov exponent diagram of the system (Eq.8) driven by a sine wave force for three values of Γ = 0.008, 0.25 and 0.5 respectively. Table 2 summarizes the numerical simulations of the system (Eq.8) as a function of parameter f for three fixed values of Γ. From this table, we note that in the f intervals periodic behaviour is recovered for f values above certain threshold values as in the previous case (ie. $\Gamma = 0$). As can be seen from Fig.2, the suppression of chaos occurs (indicated by $\lambda = 0$) when the perturbation parameter Γ is introduced in the system (Eq.8). When the parameter Γ increases from small value, the chaotic orbit is significantly reduced, this is clearly seen in Fig.2(a-c). For example, the periodic regions of the system without Γ (ie. $\Gamma = 0$) occur at (0.25, 1.25), (1.71, 2.5), (3.1, 5) whereas in the system with Γ (ie. $\Gamma \neq 0$) the periodic regions occur at (1.5, 4), (4.1, 5) for $\Gamma = 0.5$. Phase portrait is a geometric representation of the orbits of a dynamical system in a phase plane, which is drawn between position (x) and velocity (y (= $\dot{x}$)).

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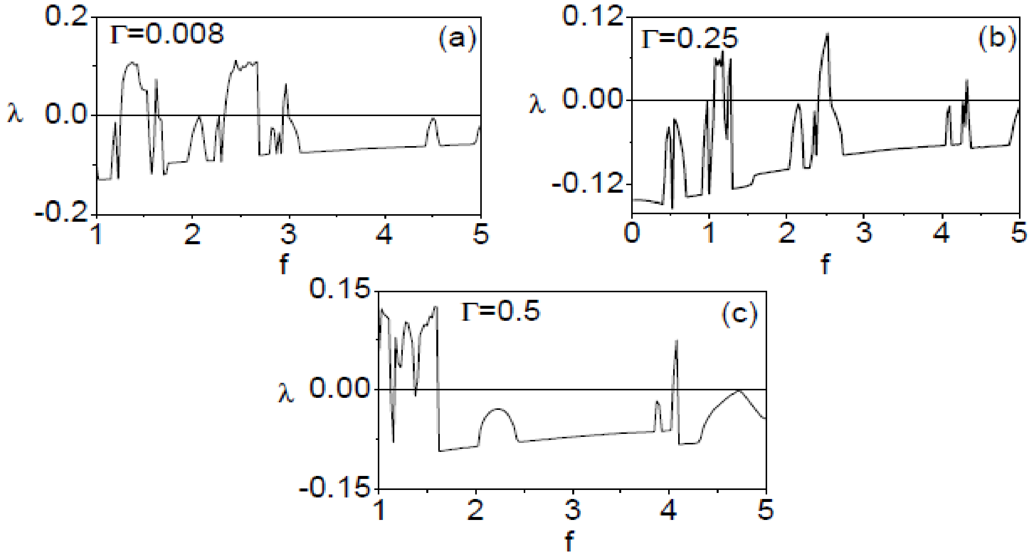

Fig. 2. Lyapunov exponents λ versus the parameter *f* for the system (Eq.8) driven by a periodic sine wave force for three values of *Γ*. The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4 and ω = 1.0

Table 2

Details of the dynamical behaviour of the system (Eq.8) in the presence of sine wave force for three fixed values of Γ as a function of *f*

value of *Γ*	Range of *f*	Dynamical behaviour
0.008	(0.0–1.25);(1.75–2.5);(2.75–3.0);(3.1–5)	periodic orbits
	(1.25–1.75);(2.5–2.75);(2.75–3.1)	chaotic orbits
0.25	(0.0–1.1);(1.25–2.5);(2.65–4.25);(4.3–5.0)	periodic orbits
	(1.1–1.25);(2.5–2.65);(4.25–4.3)	chaotic orbit
0.5	(1.5–4.0);(4.1–5.0)	periodic orbits
	(0.0–1.5);(4.0–4.1)	chaotic orbits

The trajectory of a point in a phase space represents how the state of a dynamical system changes over time, which is drawn between time (t) (in secs) and position (x). To understand the dynamics of the system with sine wave force, we studied the phase portrait and the corresponding Poincar´e map for two values of *f* chosen in the chaotic and periodic regions in Fig.2(c). Figure 3 shows the phase portraits and the corresponding Poincar´e maps at *f* = 1.5 and 4.0 with *Γ* = 0.5. From these figures we can clearly confirm the occurrence of the chaotic and period−2T orbits in the system. Time evolution of the system (Eq.8) driven by a sine wave force at *f* = 1.5 and 4.0 are shown in Fig.4. In Fig. 4(a) irregular oscillations occur at *f* = 1.5 where as periodic oscillations occur at *f* = 4.0 in Fig.4(b).

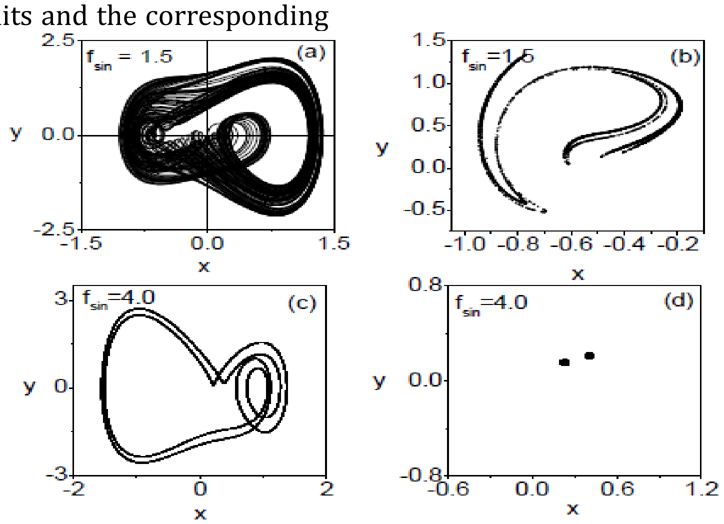

Fig. 3. Phase portraits and the corresponding Poincar´e maps for the system (Eq.8) driven by a periodic sine wave force for two values of *f* chosen in the chaotic and periodic regions in Fig.2(c). The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4, Γ = 0.5 and ω = 1.0

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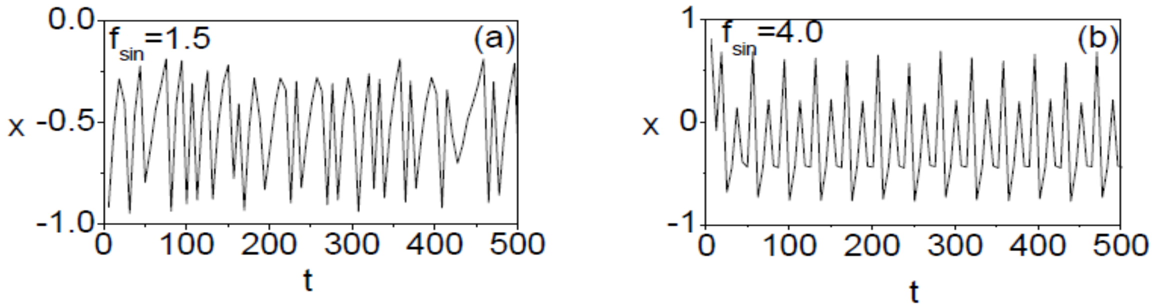

Fig. 4. Time evolution of the system (Eq.8) is driven by a periodic sine wave force for two values of *f* chosen in the chaotic and periodic regions in Fig.2(c). The values of the other parameters of the system fixed as $\alpha = -1$, $\beta = 5.0$, $\mu = 0.4$, $\Gamma = 0.5$ and $\omega = 1.0$.

(b) Effect of modulus of sine wave force

When the sine wave force is replaced by a modulus of sine wave force, suppression of chaos is also achieved in the system (Eq.8) which is clearly evident in Fig.5. When the external forcing in the system (Eq.8) is a modulus of sine wave force and if Γ = 0.008, the system has a leading Lyapunov exponent $\lambda > 0$ in the range $4.1 < f < 4.5$ and $4.75 < f < 5$, this is clearly seen in Fig.5(a). Now if Γ = 0.25 and Γ = 0.5, the leading Lyanpunov exponent $\lambda > 0$ is shown in Fig.5(b) and 5(c).

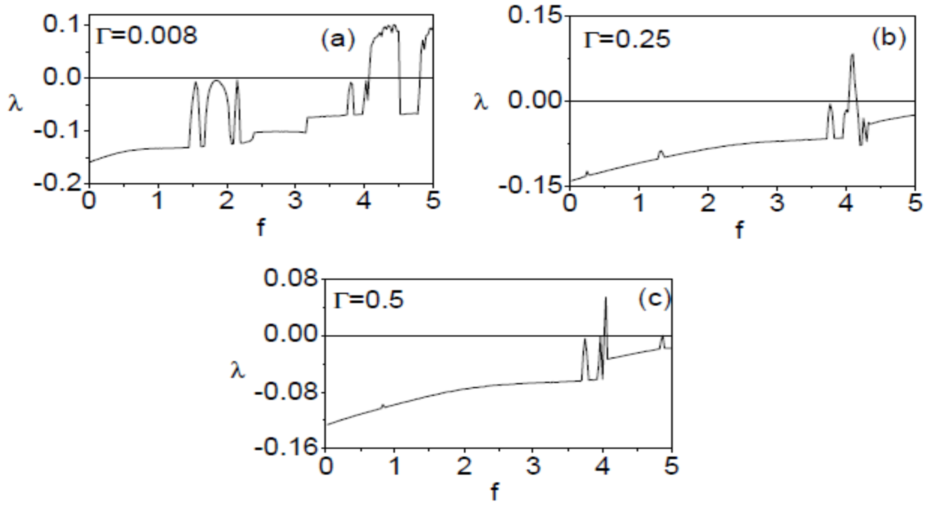

Fig. 5. Lyapunov exponents λ versus the parameter *f* for the system (Eq.8) driven by a periodic modulus of sine wave force for three values of Γ. The values of the other parameters of the system fixed as $\alpha = -1$, $\beta = 5.0$, $\mu = 0.4$ and $\omega = 1.0$

Table 3 summarizes the numerical simulations of the system (Eq.8) as a function of parameter *f* for three fixed values of Γ. From this table, we note that the regular behaviour occurs in the wider region of *f*. One thus notices that regular behaviour over a wider range of *f* occurs for three values of Γ. So with an optimal choice of the strength of Γ, suitable regular motion can be achieved in appropriate intervals of *f*. Figure 6 shows the phase portraits and the corresponding Poincar´e maps at *f* = 4.1 and 1.0 with Γ = 0.25. From these figures we can clearly confirm the occurrence of the chaotic and period−*T* orbits in the system. Time evolution of the system (Eq.8) driven by a modulus sine wave force at *f* = 4.1 and 1.0 is shown in Fig.7. In Fig. 7(a) irregular oscillations occur at *f* = 4.1 whereas periodic oscillations occur at *f* = 1.0 in Fig.7(c).

Table 3

The dynamics of the system (Eq.8) in the presence of modulus of sine wave for three fixed values of Γ as a function of *f*

value of Γ	Range of *f*	Dynamical behaviour
0.008	(0.0-4.0);(4.5-4.75)	periodic orbits
	(4.0-4.3)	chaotic orbits
0.25	(0.0-4.1);(4.25-5.0)	periodic orbits
	(4.1-4.25)	chaotic orbit
0.5	(0.0-4.0);(4.1-5.0)	periodic orbits
	(4.0-4.1)	chaotic orbits

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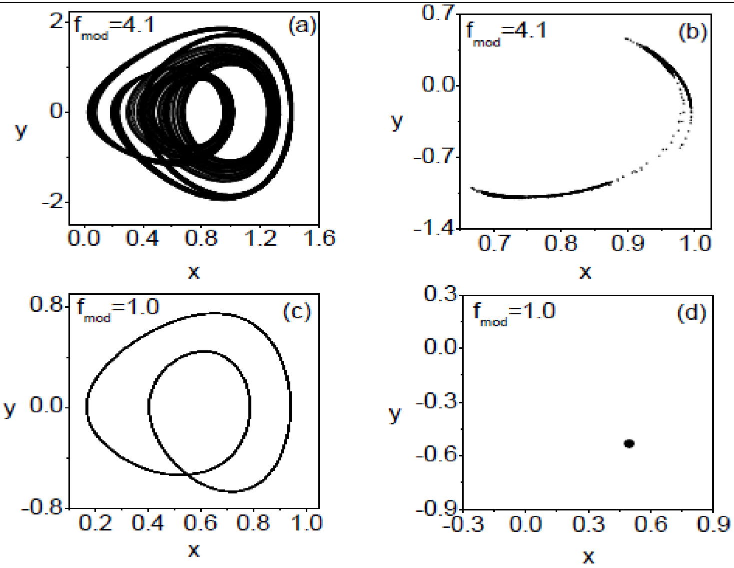

Fig. 6. Phase portraits and the corresponding Poincar´e maps for the system (Eq.8) driven by a periodic modulus of sine wave force for two values of *f* chosen in the chaotic and periodic regions in Fig.5(c). The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4, Γ = 0.5 and ω = 1.0

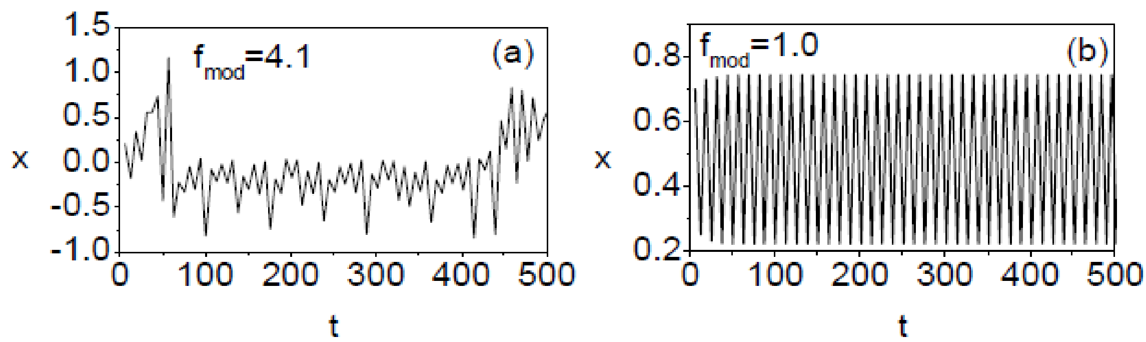

Fig. 7. Time evolution of the system (Eq.8) driven by a periodic modulus of sine wave force for two values of *f* chosen in the chaotic and periodic regions in Fig.5(c). The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4, Γ = 0.5 and ω = 1.0.

(c) Effect of rectified sine wave force

Then we study the controlling of chaos in the system (Eq.8) driven by a rectified sine wave force. Figure 8 shows the maximum Lyanpunov exponent of the modified and forced DVP oscillator with rectified sine wave force for three values of *Γ* = 0.008, 0.25 and 0.5 as the amplitude *f* is varied. The effect of *Γ* is clearly seen in Fig.8. When the parameter *Γ* increases, the region of periodic orbits also increases. Table 4 summarizes the numerical simulations of the system (Eq.8) as a function of the parameter *f* for three values of *Γ*.

Table 4

The dynamics of the system (Eq.8) in the presence of rectified sine wave for three fixed values of *Γ* as a function of *f*

value of *Γ*	Range of *f*	Dynamical behaviour
0.008	(0.0–3.25);(3.8–5.0)	periodic orbits
	(3.25–3.8)	chaotic orbits
0.25	(0.0–3.25);(4.25-4.5)	periodic orbits
	(3.25–4.25);(4.5-5.0)	chaotic orbit
0.5	(0.0–4.0)	periodic orbits
	(4.0–5.0)	chaotic orbits

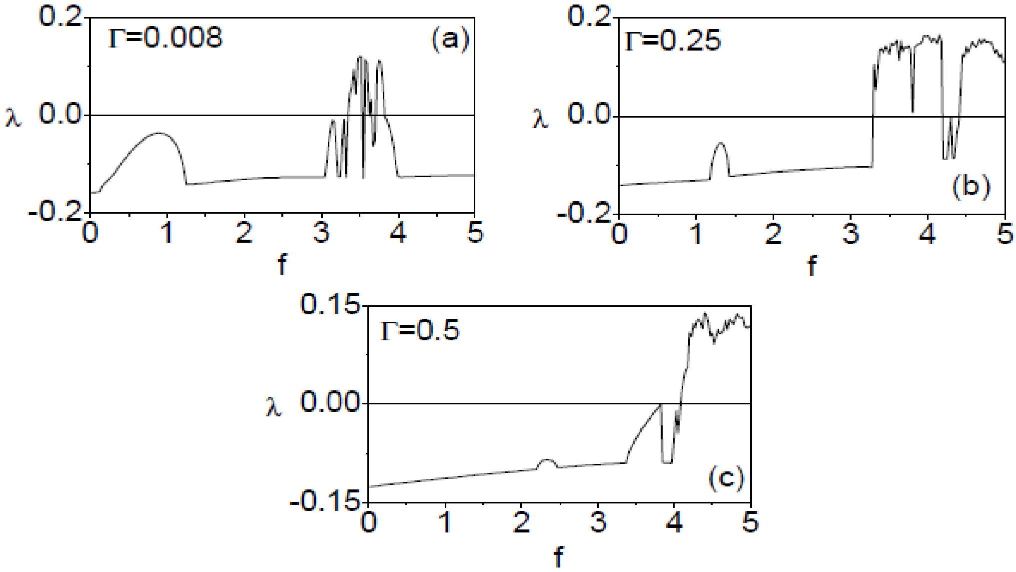

Fig. 8. Lyapunov exponents λ versus the parameter *f* for the system (Eq.8) driven by a periodic rectified sine wave force for three values of Γ. The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4 and ω = 1.0

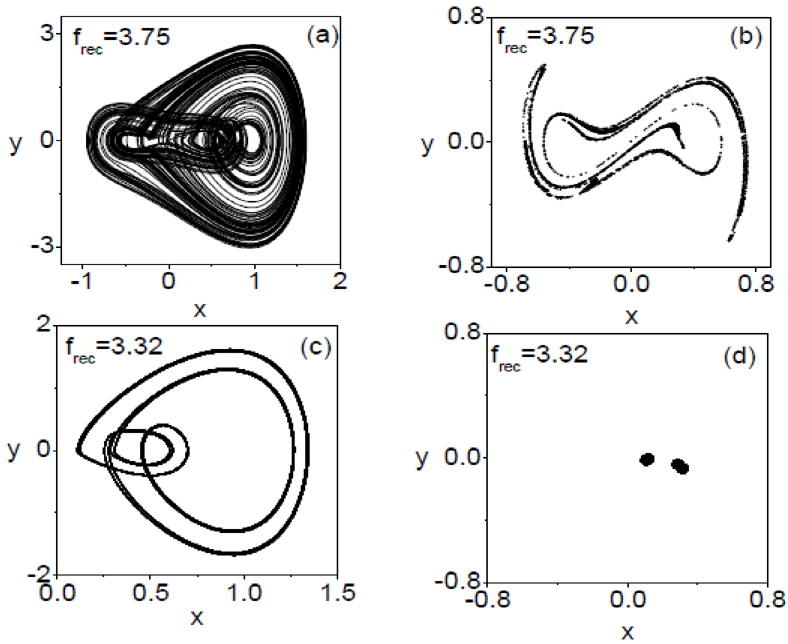

Fig. 9. Phase portraits and the corresponding Poincar´e maps for the system (Eq.8) driven by a periodic rectified sine wave force for two values of *f* chosen in the chaotic and periodic regions in Fig.8(b). The values of the other parameters of the system fixed as α = −1, β = 5.0, μ = 0.4, Γ = 0.25 and ω = 1.0

From the table 4, the effect of Γ in chaos suppression is quite obvious: periodic motions dominate a large area in the figures (Figs.8a-c). In this system, chaos and period-doubling bifurcation exist when f = 3.75 and f = 3.32, as shown in Fig.9, where Figs.9(a-c) is the phase portraits and Figs.9(b-d) is the corresponding Poincar´e maps. The above results show that a relatively weak external periodic force can significantly change the dynamics of the system.

Hysteresis phenomenon. The coexistence of several attractors gives rise to the possibility of hysteresis, that is, the possibility of jumping through the coexisting attractors in a way that is not reversible, when we fix a parameter back to its original value. It is present in the mechanical system, electromagnetism, chemical kinetics and nonlinear optics. In this section, we analyze the occurence of hysteresis phenomenon in the system (Eq.8) driven by various sinusoidal periodic forces.

First we consider the system (Eq.8) with sine wave force. We fix the parameters value as α = −1.0, β = 5.0, μ = 0.4, ω = 1.0 and Γ = 0.5. Hysteresis phenomenon is observed in the system (Eq.8) with sine wave force. The bifurcation diagram is plotted by varying f in the forward direction as well as in the reverse direction is shown in the Figs. 10(a-b).

Figures 10(a) and 10(b) are obtained by varying the amplitude *f* from 3.0 in the forward direction and from the value 5 in the reverse direction. Different paths are followed in Figs. 10(a) and 10(b). Hence the system (Eq.8) with sine wave force exhibit the hysteresis phenomenon, when the control parameter *f* is varied smoothly from a small value to a large one and then back to the smaller value. Next we analyze the existence of hysteresis phenomenon in the system (Eq.8) with modulus of sine wave force. Hysteresis is realized, when *f* is varied in the forward and reversed directions in the interval $f \in [3, 5]$ for $\Gamma = 0.5$, which is shown in Figs. 10(c)and 10(d). Finally we consider the effect of rectified sine wave force in the system (Eq.8). Hysteresis phenomenon is also realized for this force which is shown in Figs. 10(e) and 10(f)

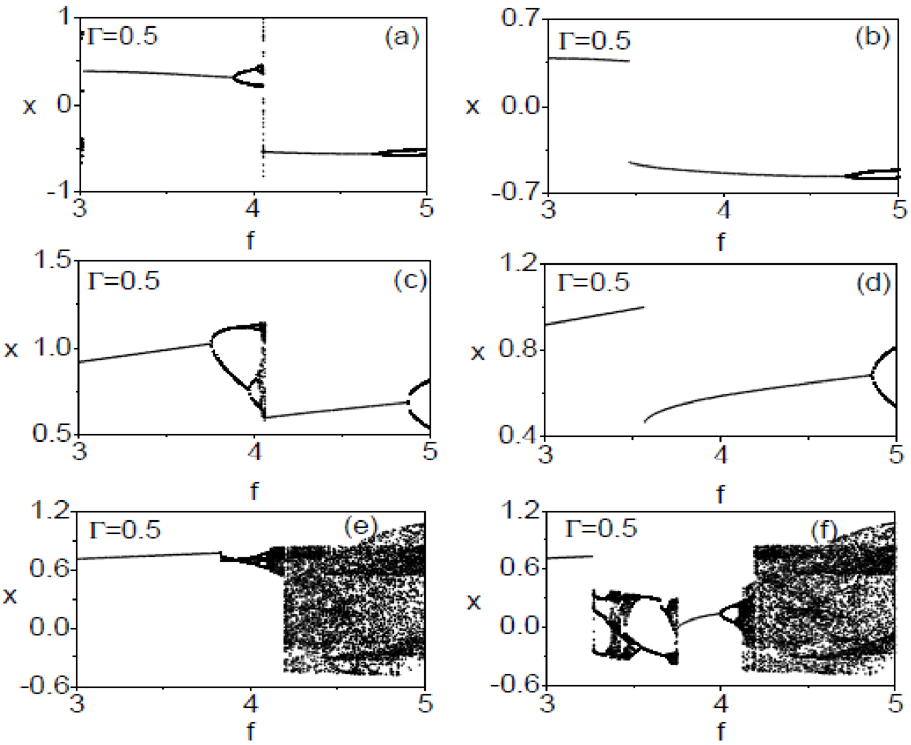

Fig. 10. Bifurcation diagrams of the system (Eq.8) driven by a (a-b) sine wave (c-d) modulus of sine wave and (e-f) rectified sine wave forces, when f is varied in the forward direction from 3 to 5 (Figs (a),(c) and (e)) and reverse direction (Figs. (b), (d) and (f)). The values of the other parameters of the system fixed as $\alpha = -1$, $\beta = 5.0$, $\mu = 0.4$, $\Gamma = 0.5$ and $\omega = 1.0$.

Conclusion

In this work, we have studied the control of chaos via external forcing in an autocatalytic dissipative chemical system governed by a forced modified Duffing – Van der Pol (DVP) oscillator. The external forces considered are sine wave, modulus of sine wave and rectified sine wave. Active control of chaos in an autocatalytic chemical system with these three forces and perturbation parameter Γ have been analyzed. From the numerical simulation made, it should be noted that for certain values of amplitude of the external forces and the perturbation parameter Γ, the chaotic behaviours can be controlled and even reduced to a periodic oscillation. The phenomenon of hysteresis and coexistence of several attractors are also obtained. The periodic and chaotic behaviours of the chemical system are analyzed through bifurcation diagram, Lyapunov exponent, time series, phase portrait and Poincar´e map.

Analytical methods such as multiple-scale perturbation and Melnikov techniques can be employed to the system to investigate certain nonlinear behaviours such as vibrational resonance, stochastic resonance, homoclinic chaos, hysteresis, chaos, etc. These will be studied in future.

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