Journal of Chemistry and Technologies, 2023, *31*(1), 195-207

Journal of Chemistry and Technologies

pISSN 2663-2934 (Print), ISSN 2663-2942 (Online).

journal homepage: http://chemistry.dnu.dp.ua

UDC 663.81

TEMPERATURE WAVES METHOD FOR MODELING REGENERATIVE HEAT EXCHANGERS

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Received 22 December 2022; accepted 30 January 2023; available online 25 April 2023

Abstract

Regenerative heat exchangers have a large surface area per unit volume and low cost, compared with other types of heat exchangers. The complexity of their proper design and optimization is one of the reasons why these devices are not widely used. The article describes a temperature-wave approach to the modeling of heat regenerators. The verification of the novel temperature wave's model was held by the experimental data of the regenerator used in ventilation systems. The temperature waves method for computation of a heat regenerator makes it possible to take into account the influence of the following factors: the variable gas temperature at the regenerator's inlet, processes of non-stationary heat conduction in the elements of packing, the longitudinal thermal conductivity of the packing. Despite a complex mathematical apparatus used to justify the method of temperature waves for designing regenerators, the very procedure for calculating such a heat exchanger has proven to be relatively simple and convenient for computer calculations.

Keywords: regenerative heat exchangers; heat regenerators; modeling; air-to-air heat exchanger.

МЕТОД ТЕМПЕРАТУРНИХ ХВИЛЬ ДЛЯ МОДЕЛЮВАННЯ РЕГЕНЕРАТИВНИХ ТЕПЛООБМІННИКІВ

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Анотація

Регенеративні теплообмінники мають велику площу поверхні на одиницю об'єму і низьку вартість, у порівнянні з іншими типами теплообмінників. Складність їх правильного проектування та оптимізації є однією з причин, чому ці пристрої не набули широкого використання. У статті описано температурно-хвильовий підхід до моделювання регенеративних теплообмінників. Перевірка нової моделі температурних хвиль проводилася по експериментальним даним для регенератора, який використовується в системах вентиляції. Метод температурних хвиль для розрахунку теплового регенератора дозволяє врахувати вплив таких факторів: змінна температура газу на вході в регенератор, процеси нестаціонарної теплопровідності в елементах насадки, поздовжня теплопровідність насадки. Незважаючи на складний математичний апарат, використаний для обґрунтування методу температурних хвиль для проектування регенераторів, сама процедура розрахунку такого теплообмінника виявилася відносно простою і зручною для комп'ютерних розрахунків.

Ключові слова: регенеративні теплообмінники; регенератори тепла; моделювання; теплообмінник повітря-повітря.

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© 2023 Oles Honchar Dnipro National University;
doi: 10.15421/jchemtech.v31i1.269522

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Introduction

Regenerative heat exchangers or heat regenerators are batch heat exchangers in which heat is first transferred from a hot gas to a high heat capacity packing. Next this heat is transferred from the packing to a cold gas. Thus, the hot and cold gases are alternately in contact with the solid material forming the packed bed. These two phases are the reason why the regenerative heat exchangers must operate in pairs to work continuously.

Regenerative heat exchangers have numerous applications in the industry, from very high temperatures to cryogenic conditions. However, the complexity of the calculations and optimization of regenerative heat exchangers limited their expansion.

The simplest mathematical model in heat regenerators proposed by Nusselt [1] has also been used by Hausen [2; 3]. This model is based on the following assumptions:

- ✓ Thermal and physical properties of the gas and the solid packing are constant and independent of temperature and position.
- ✓ The mass flow rates and the heat transfer coefficients are constant.
- ✓ The heat conductivity of packing is infinitely large in the direction normal to the gas flow and infinitely small in the direction parallel to the gas flow.
- ✓ The heat transfer in the gas is negligible in the longitudinal and transverse directions.
- ✓ Radiation heat transfer is small in comparison to the other mechanisms of heat transfer.

If we accept the above assumptions, then the operation of the regenerative heat exchanger will be described by a system of linear differential equations with constant coefficients. When cold and hot gas flows have the same mass flow rates, this system of equations has an analytical solution.

The nonlinear model considers the change in gas and bed properties as a function of temperature. The heat transfer coefficients are then calculated from these properties at every moment and place. This is important for the more realistic simulation of high-temperature regenerators. The calculation of a nonlinear model is much more time-consuming, but this is not a considerable problem with modern computers.

When the assumption of negligible axial heat conductivity in the solid packing is loosen the heat conduction mechanism in the packing in the direction of flow must be considered. This effect is shown to be significant for the bed made from metallic or thick ceramic walls [4–6].

If the calculation takes into account the longitudinal thermal conductivity of the heat exchanger packing, then such a heat exchanger must be considered as a whole. Therefore, currently widely used calculation methods based on the splitting of the heat exchanger into many sections require additional coordination of the boundary conditions at the ends of each of these sections. In this case, it is necessary to control the heat balance of the entire wall or packing of the heat exchanger, which requires consideration of the heat exchanger as one whole [7].

The most important assumption in the Nusselt-Hausen model is that the thermal conductivity of the packing material is infinite in a direction perpendicular to gas flow and zero in a direction parallel to the gas flow. It means that the packing is isothermal in a direction perpendicular to gas flow. This is approximately true where the packing is thin or is made of materials of high heat conductivity.

However, if the packing of the regenerator is constructed of material of low thermal conductivity, then it is necessary to take into account the resistance to heat transfer in the solid elements of packing.

If, in addition to the longitudinal thermal conductivity of the heat exchanger, we try to take into account non-stationary heat transfer in the elements of packing, then the difficulties in designing such devices grow exponentially.

Therefore, to consider the longitudinal thermal conductivity of the heat exchangers, especially in non-stationary heat transfer processes, it is necessary to develop fundamentally new methods and approaches.

Theoretical substantiation of the method of temperature waves for modeling periodic heat transfer processes. Let us consider the heat transfer in the heat regenerator, using the wave approach to modeling such apparatus.

For a mathematical description of non-stationary heat transfer, we will adopt the following physical pattern of heat propagation in the heat regenerator.

The entire space of the regenerator is mentally separated into two zones: a fixed packing, consisting of solid elements with stagnant zones adjacent to them, and a system of channels and voids between the elements of packing, in which the gas moves.

In the packing, heat propagation occurs mainly due to the thermal conductivity of the contacting elements of the packing.

In the system of channels and voids penetrating the packing, heat is transferred by convection.

Thus, heat in the regenerator is transferred along two separate paths, each of which has its own mechanism of heat transfer [8; 10]. The interaction between these heat flows occurs along the border of channels and voids in which the coolant moves.

With the assumptions defined in this model, the heat removal from the packing to the moving gas is considered a heat sink in the problem of non-stationary thermal conductivity of the packing. The heat transfer between the fluid stream and the solid surface is described heat balance equation over some increment distance.

Taking into account the above assumptions, the two-dimensional differential equations of unsteady heat conduction in the regenerators packing can be written as [8; 10].

$$\lambda_x \frac{\partial^2 T(x,y,\tau)}{\partial x^2} + \lambda_y \frac{\partial^2 T(x,y,\tau)}{\partial y^2} - \alpha F\left[T(x,\delta,\tau) - Tg(x,\tau)\right] = \rho_{pac} C_{pac} \frac{\partial T(x,y,\tau)}{\partial \tau}, \qquad (1)$$

where $T(x,y,\tau)$ is the current temperature of the heat exchanger packing, [K]; $Tg(x,\tau)$ – current temperature of the moving coolant, [K]; α is the coefficient of heat transfer from the surface of the packing element to the coolant flowing around it, [$W/(K\,m^2)$]; F is the surface area per unit volume of heat exchanger, [m^2/m^3]; λ_x and λ_y – equivalent value of the thermal conductivity coefficient, along the x and y coordinate, taking into account the relative volume of voids in the packing, the number of thermal contact points, etc., [W/(m K)]; ρ_{pac} – bulk density of the heat exchanger packing, [kg/m^3]; C_{pac} is the heat capacity of the heat exchanger packing, [J/(kg K)]; δ is the characteristic size of the heat exchanger packing element, [m].

The heat conduction equation of the regenerative heat exchanger packing takes into account the heat distribution in two coordinates. Coordinate x is directed along the heat exchanger axis and coincides with the direction of the coolant flow in the heat exchanger. The generalized y coordinate is orthogonal to the x and is directed along the normal to the surface of the packing element.

The heat balance of the elementary coolant volume moving through the system of cavities between the packing elements can be written as a first-order differential equation:

$$\alpha F\left[T(x,\delta,\tau) - Tg(x,\tau)\right] - GC_p \cdot \frac{dTg(x,\tau)}{dx} = 0, \qquad (2)$$

where G is the mass flow rate of the coolant, referred to a unit section of the regenerative heat exchanger, [$kg/(s\cdot m^2)$]; C_p is the isobaric heat capacity of the coolant, [J/(kg·K)].

When these two equations are considered together, they enable us to find the temperature distribution in the moving coolant and the packing itself.

After transitioning to time operator form [11], assumes the equation:

$$\begin{aligned} &\lambda_x \frac{\partial^2 \tilde{T}(x,y,s)}{\partial x^2} + \lambda_y \frac{\partial^2 \tilde{T}(x,y,s)}{\partial y^2} - \alpha F\left[\tilde{T}(x,\delta,s) - \tilde{T}g(x,s)\right] = \\ &= \rho_{pac} C_{pac}\left[s \cdot \tilde{T}(x,y,s) - To(x,y)\right], \end{aligned} \qquad (3)$$

where s is time differentiation operator, [1/s]; $To(x,y)$ initial temperature distribution in the packing, [K]; δ haracteristic size of the packing elements (radius, half thickness), [m].

For simplicity, hereinafter the initial temperature distribution will be set to zero.

The finite integral transform [12; 13] along the y coordinate is performed using the kernel

$$\overline{T}_n(x,s) = \int_0^{\delta} \tilde{T}(x,y,s) \cos\left(\mu_n \frac{y}{\delta}\right) dy; \qquad n = 0,1,2\ldots \qquad (4)$$

where μ_n is n-th eigenvalue of the integral transformation along the y-coordinate.

The inverse integral transform with respect to y

$$\tilde{T}(x,y,s)=\sum_{n=0}^{\infty}\frac{\overline{T}_n(x,s)}{\left\|\psi_{y,n}\right\|^2}\cos\left(\mu_n\frac{y}{\delta}\right);\tag{5}$$

$$\left\|\psi_{y,n}\right\|^2=\int_0^{\delta}\cos^2\left(\mu_n\frac{y}{\delta}\right)dy=\frac{\delta}{2}+\frac{\delta}{4\mu_n}\sin(2\mu_n);\tag{6}$$

where $\left\|\psi_{y,n}\right\|^2$ *n*-th normalizing factor (square of the norm of the kernel of the integral transformation) along the y-coordinate, [m].

The eigenvalues of the integral transform are found such that type-3 boundary conditions are satisfied on the packing element surface, and that the type-2 zero boundary condition is satisfied on the packing element's axis of symmetry. The transcendental equation for finding the eigenvalues in this case is

$$\left(\frac{\mu_n}{\delta}\right)\cdot tg(\mu_n)=\frac{\alpha}{\lambda_y}.\tag{7}$$

After the cosine transform along the y coordinate, the heat equation in fixed packing assumes the form

$$\lambda_x\frac{\partial^2\overline{T}_n(x,s)}{\partial x^2}-\lambda_y\frac{\mu_n^{\;2}}{\delta^2}\overline{T}_n(x,s)-\alpha F\frac{\mu_n}{\delta}\sin(\mu_n)\left[\overline{T}_n(x,s)-\overline{Tg}_k(x,s)\right]=$$
$$=\rho_{pac}C_{pac}s\cdot\overline{T}_n(x,s);\tag{8}$$

The integral transform along the *x* coordinate is performed after choosing cosine function as the transform kernel

$$\overline{\overline{T}}_{n,k}(s)=\int_0^h\overline{T}_n(x,s)\cos\left(\pi\cdot k\frac{x}{h}\right)dx\ ;\ \ k=0,1,2\ldots\tag{9}$$

The choice of this type of integral transform kernel automatically ensures the type-2 zero boundary conditions at the ends of the regenerative heat exchanger; in other words, there is no heat exchange along the x coordinate at the ends of the heat exchanger.

After carrying out the final integral transform with respect to the x coordinate, Eq. (8) assumes the form

$$-\lambda_x\frac{(\pi\cdot k)^2}{h^2}\overline{\overline{T}}_{n,k}(s)-\lambda_y\frac{\mu_n^{\;2}}{\delta^2}\overline{\overline{T}}_{n,k}(s)-\alpha F\frac{\mu_n}{\delta}\sin(\mu_n)\left[\overline{\overline{T}}_{n,k}(s)-\overline{Tg}_k(s)\right]=$$
$$=\rho_{pac}C_{pac}s\cdot\overline{\overline{T}}_{n,k}(s);\tag{10}$$

From here, we find the function that describes the temperature of the fixed packing,

$$\overline{\overline{T}}_{n,k}(s)=\frac{\overline{Tg}_k(s)\dfrac{\alpha F}{\rho_н C_н}\dfrac{\mu_n}{\delta}\sin(\mu_n)}{\left(a_x\dfrac{(\pi\cdot k)^2}{h^2}+a_y\dfrac{\mu_n^{\;2}}{\delta^2}+\dfrac{\alpha\cdot F}{\rho_{pac}C_{pac}}\dfrac{\mu_n}{\delta}\sin(\mu_n)+s\right)},\tag{11}$$

where a_x and a_y is packing layer thermal diffusivity, [m^2/s]. The obtained equality allows us to find the solution to the packing heat equation, in the form of a double Fourier series:

$$\tilde{T}(x,y,s)=\sum_n\sum_k\frac{\overline{Tg}_k(s)\dfrac{\alpha F}{\rho_н C_н}\cdot\dfrac{\mu_n}{\delta}\sin(\mu_n)\cos\left(\mu_n\dfrac{y}{\delta}\right)\cdot\cos\left(2\pi\cdot k\dfrac{x}{h}\right)}{\left(a_x\dfrac{(\pi\cdot k)^2}{h^2}+a_y\dfrac{\mu_n^{\;2}}{\delta^2}+\dfrac{\alpha F}{\rho_{pac}C_{pac}}\dfrac{\mu_n}{\delta}\sin(\mu_n)+s\right)\cdot\left\|\psi_{x,k}\right\|^2\cdot\left\|\psi_{y,n}\right\|^2}.\tag{12}$$

Were the coefficients $\overline{Tg}_k(s)$ do not depend on coordinates and are found according to the formula

$$\overline{Tg}_k(s)=\int_0^h\overline{Tg}_k(x,s)\cdot\cos\left(2\pi\cdot k\frac{x}{h}\right)dx.\tag{13}$$

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Until now, the solution to the heat equation for the packing was obtained via finite integral transformations method, as described in [12; 13] for example.

We introduce the concept of an eigen temperature wave for a given regenerative heat exchanger.

If the temperature of a coolant flow periodically changes then it may be expanded into a series of harmonic temperature fluctuations of the moving coolant. Each of these abstract objects, in which the coolant temperature changes according to a harmonic law, we will agree to call a temperature wave.

This object is called a temperature wave because for a stationary observer, passing by a coolant flow, the temperature of which changes according to a harmonic law, looks like the passage of a temperature wave.

It is necessary to note the fundamental difference between the described temperature wave and sound or electromagnetic waves. Sound and electromagnetic waves exist in nature, and temperature waves are abstract objects used in regenerative heat exchanger calculations.

If the steady operating mode of a regenerative heat exchanger is considered, then according to Prigogine's theorem, the entropy production in such a heat exchanger should reach its minimum.

It is obvious that the absolute minimum of entropy production in the steady process of the temperature wave's passage through the packing of a regenerative heat exchanger will be achieved only when the entropy of the packing as a whole remains unchanged. And this mode of infinite temperature wave transmission through the packing of the regenerator, at which the packing entropy remains unchanged, does exist. This mode takes place at the moving coolant's temperature fluctuation frequencies, for which the instant value of its temperature at the entrance to the packing nozzle is equal to the instant temperature of this coolant at the packing nozzle exit. This is possible only when an integer number of temperature waves fitted inside the heat exchanger packing.

It should be noted that the heat transfer at a finite temperature difference is always accompanied by an increase in entropy. Therefore, the work of any heat exchanger is accompanied by the production of entropy. But in this case, the mode with zero entropy production is realized not in the heat exchanger, but in the packing through which the infinite eigen temperature wave passes.

As the steady temperature eigenwave moves through the heat regenerator packing, and the heat inside the packing itself is redistributed, but the energy of the heat accumulated in the packing, remains unchanged.

Let's clarify the wave nature of the coolant temperature fluctuations. Not by arbitrary oscillation frequencies, but by eigenfrequencies for the given regenerative heat exchanger, i.e. such frequencies that a whole number of temperature waves fit along the length of the regenerative heat exchanger:

$$\begin{aligned} Tg_k(x,\tau) &= B_k \cos\left(2\pi \cdot k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right) = \\ &= B_k\left[\cos\left(2\pi \cdot k\frac{x}{h}\right)\cos\left(2\pi \cdot k\frac{\tau}{T}\right)-\sin\left(2\pi \cdot k\frac{x}{h}\right)\sin\left(2\pi \cdot k\cdot\frac{\tau}{T}\right)\right], \end{aligned} \quad (14)$$

where B_k amplitude of temperature fluctuations in the k-th harmonic of a temperature wave moving in a coolant, [K]; h is packing layer height in a regenerative heat exchanger, [m]; T is period of operation of the regenerative heat exchanger (period of temperature fluctuations in the main temperature wave), [s].

We build the finite integral cosine transform with respect to coordinate x:

$$\begin{aligned} \overline{Tg}_{m,k}(\tau) &= \int_0^h Tg_k(x,\tau)\cos\left(2\pi \cdot m\frac{x}{h}\right)dx = \\ &= B_k \cos\left(2\pi \cdot k\cdot\frac{\tau}{T}\right)\int_0^h \cos\left(2\pi \cdot k\frac{x}{h}\right)\cos\left(2\pi \cdot m\frac{x}{h}\right)dx - \\ &- B_k \sin\left(2\pi \cdot k\cdot\frac{\tau}{T}\right)\int_0^h \cos\left(2\pi \cdot k\frac{x}{h}\right)\sin\left(2\pi \cdot m\frac{x}{h}\right)dx. \end{aligned} \quad (15)$$

To find the values of the obtained integrals, we use the frequency selection rule:

$$\int_0^h \cos\left(2\pi\cdot k\frac{x}{h}\right)\cos\left(2\pi\cdot m\frac{x}{h}\right)dx=\begin{Bmatrix} m=k\Rightarrow \dfrac{h}{2} \\ m\neq k\Rightarrow 0\end{Bmatrix}$$
$$\int_0^h \cos\left(2\pi\cdot k\frac{x}{h}\right)\sin\left(2\pi\cdot m\frac{x}{h}\right)dx=0 \tag{16}$$

With this in mind

$$\overline{Tg}_k(\tau)=B_k\cos\left(2\pi\cdot k\cdot\frac{\tau}{T}\right)\frac{h}{2}. \tag{17}$$

We transition to time operator form:

$$\overline{Tg}_k(s)=\frac{B_k\cdot h}{2}\frac{s}{\left(s^2+\left(\dfrac{2\pi\cdot k}{T}\right)^2\right)}. \tag{18}$$

The reasoning behind the last transform is that if a coolant passes through the heat exchanger packing and its temperature changes harmonically with a frequency equal to the frequency of one of the regenerative heat exchanger's eigen temperature waves, then all Fourier series Eq. (12) coefficients, except one, will be zero. Consequently, the solution to the heat equation of the packing in this case can be simplified and presented as a simple Fourier series, instead of a double one Eq. (12).

As is known from oscillation theory, the frequency of the forced oscillations of a system always coincides with the frequency of the external influence that causes these oscillations. Therefore, the frequency of the temperature fluctuations inside the packing element will coincide with the frequency of the temperature wave fluctuations supplied from the outside. In our case, this will be one of the eigen frequencies of the considered regenerative heat exchanger. In general, the length of the temperature waves propagating in the packing element differs from the length of the eigen temperature waves through the regenerative heat exchanger.

As a result, if a k-eigen temperature wave is supplied to the input of a regenerative heat exchanger, then in the steady state, we obtain the following expression for the form of the temperature wave in the packing:

$$\tilde{T}_{n,k}(x,y,s)=\frac{B_k\cdot h}{2}\frac{\alpha\cdot F}{\rho_{pac}C_{pac}}\frac{\cos\left(2\pi\cdot k\dfrac{x}{h}\right)\cdot s}{\left(s^2+\left(\dfrac{2\pi\cdot k}{T}\right)^2\right)}\times$$
$$\times\sum_n\frac{\dfrac{\mu_n}{\delta\cdot\left\|\psi_{y,n}\right\|^2}\sin(\mu_n)\cos\left(\mu_n\dfrac{y}{\delta}\right)}{\left(a_x\dfrac{(\pi\cdot k)^2}{h^2}+a_y\dfrac{\mu_n{}^2}{\delta^2}+\dfrac{\alpha\cdot F}{\rho_{pac}C_{pac}}\dfrac{\mu_n}{\delta}\sin(\mu_n)+s\right)}. \tag{19}$$

We introduce the following notation, in order to simplify:

$$Da_k=\frac{\mu_k\sin(\mu_k)}{\delta\cdot\left\|\psi_{y,n}\right\|^2}; \tag{20}$$

$$Dc_k=\left(a_x\frac{(\pi\cdot k)^2}{h^2}+a_y\frac{\mu_k{}^2}{\delta^2}+\frac{\alpha\cdot F}{\rho_{pac}C_{pac}}\frac{\mu_k}{\delta}\sin(\mu_k)\right); \tag{21}$$

$$Ds_k=\left(\frac{2\pi\cdot k}{T}\right). \tag{22}$$

With this in mind, Eq. (19) assumes the following form

$$\tilde{T}_{n,k}(x,y,s)=\frac{B_k\cdot h}{2}\frac{\alpha\cdot F}{\rho_{pac}C_{pac}}\cos\left(2\pi\cdot k\frac{x}{h}\right)\sum_n\frac{Da_n\cos\left(\mu_n\dfrac{y}{\delta}\right)}{\left(Dc_{k,n}+s\right)}\frac{s}{\left(s^2+Ds_k{}^2\right)}. \tag{23}$$

Expanding the obtained expression into simple fractions, we move from the result to the original with respect to time. Having dropped the terms that correspond to the transition process, we obtain a solution for the steady process of the k-eigen temperature wave passing through the packing of the regenerative heat exchanger:

$$T_k(x,y,\tau)=\frac{B_k\cdot h}{2}\frac{\alpha\cdot F}{\rho_{pac}C_{pac}}\cos\left(2\pi\cdot k\frac{x}{h}\right)\times$$
$$\times\sum_n\frac{Da_{k,n}\cos\left(\mu_n\frac{y}{\delta}\right)}{\left[Dc_{k,n}{}^2+Ds_k{}^2\right]}\left\{Ds_k\sin\left[\left(\frac{\pi\cdot k}{T}\right)\tau\right]+Dc_{k,n}\cos\left[\left(\frac{2\pi\cdot k}{T}\right)\tau\right]\right\}. \tag{24}$$

If the value of the y coordinate y is fixed, then the obtained solution can be interpreted as the result of a fixed time delay of the temperature wave propagating in the elements of heat exchanger packing. For example, on the surface of the packing elements at y =δ, the equation can be written in a form that is more convenient for analysis

$$T_k(x,\delta,\tau)=B_k\frac{\alpha F}{\rho_{pac}C_{pac}}\cos\left(2\pi\cdot k\frac{x}{h}\right)\cos(\mu_k)\left\{\tilde{D}s_k\sin\left[\left(\frac{\pi\cdot k}{T}\right)\tau\right]+\tilde{D}c_k\cos\left[\left(\frac{2\pi\cdot k}{T}\right)\tau\right]\right\}, \tag{25}$$

where $\tilde{D}s_k=\sum_n\frac{Da_n\cos(\mu_n)}{Dc_{k,n}{}^2+Ds_k{}^2}Ds_k$; $\tilde{D}c_k=\sum_n\frac{Da_n\cos(\mu_n)}{Dc_{k,n}{}^2+Ds_k{}^2}Dc_{k,n}$.

After elementary transformations, we obtain a simpler expression that is easier to analyze

$$T_k(x,\delta,\tau)=\frac{B_k\frac{\alpha F}{\rho_{pac}C_{pac}}}{\sqrt{\tilde{D}c_k{}^2+\tilde{D}s_k{}^2}}\cos\left(2\pi\cdot k\frac{x}{h}\right)\cdot\cos\left(\left(\frac{2\pi\cdot k}{T}\right)\tau-\tilde{\varphi}_k-\mu_k\right), \tag{26}$$

$$\tilde{\varphi}_k=arctg\left(\frac{\tilde{D}s_k}{\tilde{D}c_k}\right), \tag{27}$$

where $\tilde{\varphi}_k$ is the angle of temporary phase shift of temperature waves, [rad].

The physical meaning of this formula indicates that the amplitude of the temperature fluctuations in the packing is proportional to the amplitude of the coolant temperature fluctuations. The phase of the packing temperature fluctuations lags behind the phase of the coolant temperature fluctuations at the regenerator inlet by an angle $\tilde{\varphi}_k$.

In other words, the temperature wave propagation rate through the packing of the heat exchanger is less than the speed of the temperature wave outside the packing, see Fig. 1.

The heat balance of the coolant's elementary volume as it moves inside the regenerator packing is written as differential Eq. (2), the physical meaning of which is that the changes in the temperature of the gas moving through the regenerator occur only through heat exchange with the packing.

We will look for a solution to this equation in the form of a temperature cosine wave in the coolant flowing through the heat exchanger

$$Tg_k(x,\tau)=B_k\cos\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right], \tag{28}$$

The heat from the coolant is only transferred to the elements of the heat exchanger packing, and vice versa heat is transferred from the packed bad to the heat carrier flowing through it. Consequently, if the temperature of the packing element rises, then the temperature of the coolant should decrease, and vice versa if the packing cools down, then the coolant heats up. To satisfy these conditions, we represent the temperature wave on the surface of the elements of packing in the form of a sine wave

$$T_k(x,\delta,\tau)=Ab_k\cdot\sin\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right], \tag{29}$$

where Ab_k is amplitude of the k-th temperature wave in the packing, [K].

After substituting the functions of the selected type into the heat balance equation (2), we obtain

$$\frac{\alpha F}{GC_p}Ab_k\sin\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right]-\frac{\alpha F}{GC_p}B_k\cos\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right]=$$
$$=-\frac{2\pi k}{h}B_k\sin\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right]. \quad (30)$$

Bringing like terms and using the formula for the sine of the angle difference, we get

$$B_k\sin\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)-\phi_k\right]=$$
$$=\frac{\frac{\alpha F}{GC_p}}{\sqrt{\left(\frac{2\pi k}{h}\right)^2+\left(\frac{\alpha F}{GC_p}\right)^2}}Ab_k\sin\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)\right] \quad (31)$$

The spatial phase shift angle of the temperature fluctuations of packing element surface and coolant temperature fluctuations

$$\psi_k = arctg\left\{\frac{\alpha F\cdot h}{2\pi\cdot k\cdot GC_p}\right\}, \quad (32)$$

where ψ_k angle of spatial shift of phases of temperature waves, [rad].

Taking into account the solution obtained for the eigen value in the packing Eq. (26), as well as the conclusion that the amplitudes of the eigen temperature waves at the inlet and output of the packing are equal, we obtain a relatively simple solution for the eigen temperature waves in the coolant leaving the regenerative heat exchanger:

$$Tg_k(x,\tau)=\frac{B_k}{2}\cos\left[2\pi k\left(\frac{x}{h}+\frac{\tau}{T}\right)-\tilde{\varphi}_k-\psi_k-\mu_k\right]. \quad (33)$$

The physical meaning of this formula is entirely obvious: as the eigen wave passes through the regenerator packing, its amplitude and frequency remain unchanged, and the phase of the wave lags in time by an angle $\tilde{\varphi}_k$, and by the angle ψ_k in space, see Fig. 1. Moreover, for temperature waves of different lengths, the phase shift angles are different.

Formulas (14) and (33) are show that the propagation of a temperature wave described by absolutely the same dependence as the propagation of an ordinary wave, for example, a sound or electromagnetic wave. Just as with the propagation of electromagnetic and sound waves, the velocity of propagation of temperature waves decreases in a dense medium. The main difference between temperature waves and ordinary waves is that as a result of the superposition of sound or electromagnetic waves, their amplitudes add up, and the superposition of temperature waves occurs according to a completely different law.

When merging or mixing two coolant streams, the temperature of the combined stream will be determined as the weighted average of the temperatures of the two mixed streams.

Let's considered that when we decompose the periodically changing temperature of the coolant entering the regenerative heat exchanger into a Fourier series, each harmonic corresponds to some part of the coolant flow, and these parts of the coolant flow are the same for all harmonics. Then the amplitude of the heat carrier temperature fluctuations after mixing two flows can be found by the formula

$$\frac{B_k+B_n}{2}=\frac{B_k}{2}+\frac{B_n}{2}. \quad (34)$$

Therefore, to obtain the correct value of the amplitude of the temperature wave after summing the two harmonics, it is sufficient to add the temperature waves whose amplitudes are equal to half of their amplitudes before mixing. This explains the appearance of two in the denominator of the formula (33).

The solutions obtained make it possible to simulate the passage of an arbitrary-shaped periodic temperature signal through the heat exchanger packing.

When we build a model of a regenerative heat exchanger, it is necessary to keep in mind that in a heat regenerator, gas flow moves alternately, now in one direction, then in the other. This problem can be solved using the linearity of differential equations (1) and (2).

Journal of Chemistry and Technologies, 2023, *31*(1), 195-207

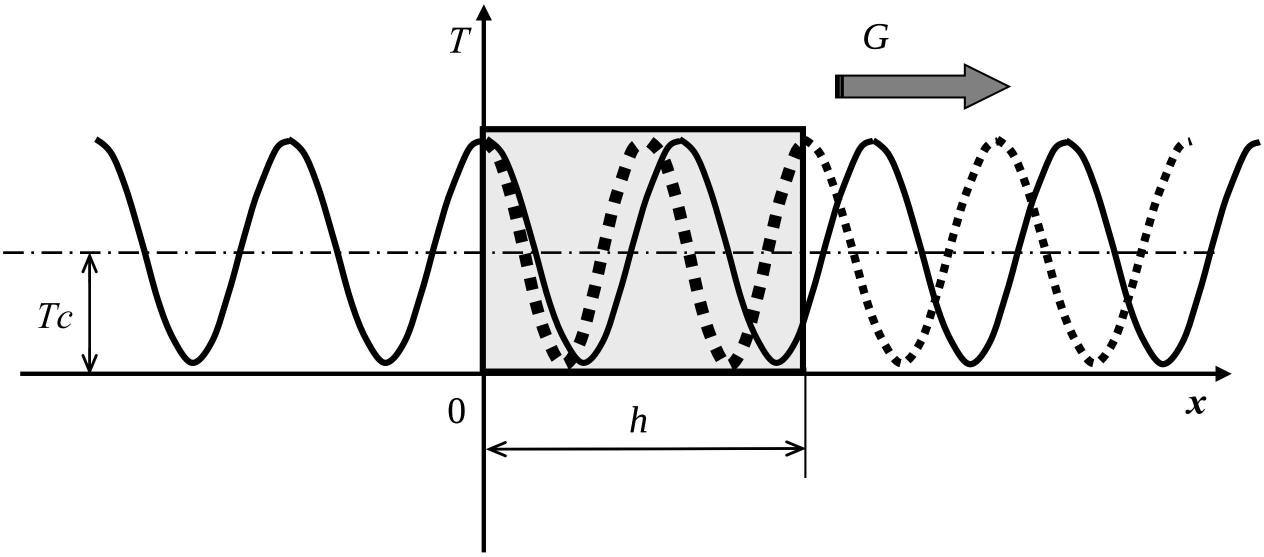

Fig. 1. Propagation of eigen temperature wave in the packing of the regenerative heat exchanger

As is well known, if there are two particular solutions to a system of linear homogeneous differential equations, then the sum or difference of these solutions will also be a solution to this system of differential equations.

Since the sum and difference of the solutions to this problem are equal from a mathematical point of view, therefore the system of differential equations (1) and (2) have two independent solutions. One of these solutions corresponds to the counterflow of heat carriers in the considered heat exchanger, and the other corresponds to the parallel movement of heat carriers. Therefore, to construct a mathematical model of the regenerative heat exchanger, in which the coolant moves in opposite directions, it is necessary to add the solutions for warm blast and cold blast.

Since the amplitude of the stationary temperature wave does not change when it passes through the packing, the heat balance of the gas and packing for the variable part of the temperature wave is performed automatically. The stationary part of the Fourier series, in which the solution to the heat transfer problem is presented, is determined from the heat balance of the heat exchanger as a whole.

Comparison of calculation results with experimental data

Let us demonstrate the application of the wave method of modeling the regenerative heat exchanger by the example of calculating a heat regenerator used to reduce heat losses in building ventilation systems.

The results of the experimental study of this type of commercial heat regenerator are given in [14].

During the experimental study, the air, during the first 41 seconds, moved through the heat exchanger packing from the building to the outside, giving off heat to the packing. For the next 41 seconds, the air passed through the packing in the opposite direction, taking back the heat from the heat regenerator packing. The change in the direction of airflow was realized by changing the rotation direction of the fan, driving air through the heat regenerator filling.

The packing of the heat exchanger was a rectangular polypropylene honeycomb, inside which air passes. The wall thickness of honeycombs is $2\delta = 0.25$ mm. The thermal conductivity of polypropylene, from which the honeycombs was produced, is ~0.19 W/(m·K). The size of the rectangular channels in the investigated heat exchanger was 1.5x3.25 mm, the length of the heat exchanger was 180 mm, and the diameter of the heat regenerator was 198 mm. The article [14] presents the results of measuring the dependence of air and packing temperatures on time at the airflow rate of 51 m^3/h.

Knowing the dimensions of the heat exchanger and the properties of airflow that passes through it, it is easy to determine the Nusselt number characterizing the heat transfer from the packing surface, Nu=4. In this case, the parameter $\alpha\delta/\lambda_y$ included in the transcendental equation (7) for finding the eigenvalues has values of the order of 0.086. Therefore, the corresponding values, found by formula (7) will be close to π·n, where n is an integer. Therefore, when calculating the sum of the Fourier series with index n, we can restrict ourselves to calculating a relatively small number of terms in this series. Only the first ten terms of the Fourier series were summed.

Experimental data for comparison was obtained at the indoor air temperature of 23 °C and the air temperature outside the building of −6.5 °C.

The paper [14] presents the results of an experimental study of air and packing temperatures for three successive cycles of regenerative heat exchanger operation. For more information, these data are shown below as repeated measurements during one period of regenerator work.

Fig. 2 shows the graphs that show the experimental values of the air temperature at the heat regenerator inlet nozzle during the warm and cold blast, taken from [14] and the results of the Boltzmann approximation of these data. The half-period of the cold blast in the regenerative heat exchanger is marked with a gray fill on the graph.

The noticeable scatter of the experimental values of the air temperature at the inlet to the regenerative heat exchanger is mainly because the measured temperature did not repeat precisely during different cycles of the heat regenerator operation.

W expands the input temperature signal into a Fourier series to build a wave model in the regenerative heat exchanger.

To determine the expansion coefficients, we use the formulas known from the mathematical analysis:

$$a_k = \int_0^T f_B(\tau)\sin\left(2\pi k\frac{\tau}{T}\right)d\tau\,;\,b_k = \int_0^T f_B(\tau)\cos\left(2\pi k\frac{\tau}{T}\right)d\tau\,, \qquad (35)$$

where $f_B(\tau)$ – the function expanded in a Fourier series in this case, it is the function obtained as a result of the Boltzmann approximation of the air temperature at the entrance to the heat regenerator.

Thus, two sets of coefficients for two Fourier series are obtained, one set of coefficients for the warm blast and another coefficient set for the cold blast.

After that, we find phase shifts for temperature waves in the airflow leaving the heat exchanger packing and for temperature waves that propagate in the packing itself, using formulas (26) and (33).

Next all the temperature waves summarized at the heat exchanger outlet nozzle according to the rule, formula (34) given.

For the reverse airflow, the similar calculation was carried out during the cold blast period. At this stage, it is possible to take into account, for example, the difference in air mass flow during the half-periods of the warm and cold blast.

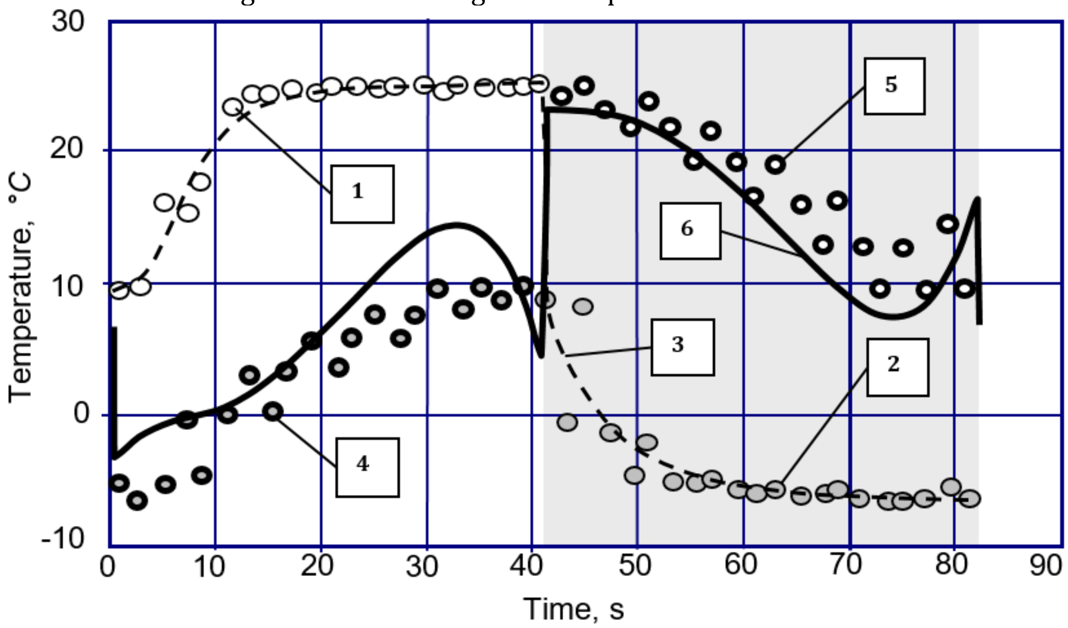

Fig. 3. Calculated values of air temperatures at the exit from the regenerative heat exchanger in the warm and cold blast half-periods and their measured values [14].
1 - air temperature at the inlet nozzle in the half-period of warm blast;
2 - air temperature at the inlet nozzle in the half-period of cold blast;
3 - Boltzmann approximation of the measurements of air temperature at the inlet to the heat exchanger;
4 - measured values of the air temperature at the exit from the heat regenerator in the half-period of warm blast;
5 - measured values of the air temperature at the exit from the heat regenerator in half-cycle of cold blast;
6 - calculated values of air temperatures at the exit from the regenerative heat exchanger in half-cycles of warm and cold blast

Since the countercurrent heat exchange scheme is implemented in the heat exchanger under consideration, the solutions obtained for warm and cold blasts are summed up.

In this example, the calculation was carried out for the first one hundred and fifty terms of the Fourier series. It allows practically excludes fluctuations in the values of the sum of the series, which usually occur when the infinite Fourier series is finished.

Fig. 3 shows the calculated values of the air temperatures at the outlet of the regenerative heat exchanger in the half-periods of a warm and cold blast and their measured values according to the data obtained from [14].

The form of the obtained graphs corresponds to the physics concepts of process in the packing. During the warm blast time, the air temperature rises at the outlet of the heat exchanger. And vice versa, during a cold blast observed the opposite pattern, the air temperature at the outlet of the regenerative heat exchanger drops.

Fig. 4 shows the calculated and experimental graphs of the packing temperature at the inlet to the heat exchanger in the half-periods of warm and cold blasts.

Graphs show good agreement between experimental and calculated data.

A comparison of graphs for air temperature, shown in Fig. 2, and temperature change of the packing at the inlet to the heat exchanger in Fig. 4, shows that they are similar. Such a result is quite expected, since the temperature of the packing, which is thin polypropylene plates, cannot much differ from the temperature of the air surrounding this packing. It is calculated, that temperature of the packing surface upon the warm blast is 0.5 °C lower than the temperature of the air. During the cold blast, the packing surface is 0.5 °C warmer than the air.

This result can serve as a test for assessing the adequacy of the wave model of heat transfer in the regenerative heat exchanger since the calculated values of the packing temperature were obtained by adding 150 temperature waves, and the amplitude and phase shift for each wave were calculated by the formula (26).

With the wave method of calculating a regenerative heat exchanger, only the most necessary information is obtained about the temperatures of the coolant and packing at the inlet and outlet of the heat exchanger in the steady-state operation. A smaller amount of information obtained when calculating the regenerator by the wave method can be regarded as a price for the relative simplicity of implementing the wave method for calculating regenerative heat exchangers.

Discussion

The graphs (see Fig. 3) show that at the end of the cold blast half-cycle, the temperature of the airflow leaving the heat exchanger slows its fall and even rises slightly. The temperature of the airflow leaving the heat exchanger during the warm blast period behaves similarly. At the end of the warm blast half-cycle, the air temperature at the outlet of the heat exchanger stabilizes. This feature of the operation of the regenerative heat exchanger is reflected in the results of calculations carried out using the wave method.

The noted feature in the course of temperatures of the airflow leaving the regenerative heat exchanger may be due to the processes of non-stationary heat transfer in the packing elements. By the end of the half-cycle of the regenerator operation, the thermal wave moving in the packing has time to exit at the other end of the heat regenerator. From time to time, we can observe this effect in the operation of regenerative heat exchangers. In our case, if the package length were longer or the heat exchanger operation period was shorter, then this effect would not be observed. Since in earlier models of the operation of a regenerative heat exchanger, non-stationary heat transfer inside the packing elements was not taken into account, nothing of the kind was noted in the calculation results from these models.

When comparing the results of calculation with the measurements results of the air temperature at the heat exchanger outlet, attention pinched to a noticeable discrepancy between the experimental and calculated data, which has a character of systematic error.

When the air temperature at the outlet of the heat exchanger increased, the results of its measurements turned out to be less than the calculated values, and if the air temperature fell, the measurement results turned out to be higher than the calculated temperature values. In addition, the greater the calculated rate of temperature change, then the larger the discrepancy between the measured and calculated values of air temperature. This character of the differences between the calculated and measured temperatures can be explained by the dynamic error of the instruments used to measure the temperature.

According to the article [14], the time required to change the direction of fan rotation in the experimental installation was about 7 seconds.

According to the graphs in Fig. 2, the airflow temperature entering the regenerative heat exchanger becomes practically equal to the air temperature in the room (23 °C) in about 12–14 seconds. And the same amount of time is required to reach the temperature outside the building (−6.5 °C) during the cold blast half-period. Therefore, in addition to the time, it takes to change the fan rotation direction, there is an additional delay of 5–7 seconds. This delay may be due to the thermal inertia of the sensor used to measure the air temperature and/or heat gains through the wires that connect this sensor to the measuring device.

Therefore, the presence of a dynamic error in measuring the air temperature can explain that part of the discrepancy between the experimental and calculated data, which is systematic.

As can be seen from the graphs, the agreement between the air temperature and the temperature of the heat exchanger packing turned out to be much better than the agreement between the calculated and experimental values for the air temperature at the exit from the heat regenerator. This can be explained by the fact that experimental values of the air temperature at the regenerator inlet were used to calculate the packing surface temperature. The measured temperature of the packing surface also included the dynamic error of temperature measurement. Since the rates of change in air temperatures at the regenerator's inlet and the packing surface are very similar, the dynamic errors in measuring their temperatures turned out to be almost the same. Therefore, the dynamic error in measuring temperature does not lead to a noticeable discrepancy between the calculated and measured temperatures of the packing surface at the inlet to the heat exchanger. The rate of change in air temperature at the outlet nozzle of the heat exchanger is significantly different from the rate of change in air temperature at the inlet nozzle. Therefore, at the exit nozzle, discrepancies between the experimental and calculated values of air temperatures are much higher.

A significant influence of the dynamic error in the measurement of rapidly changing air temperature on the results of evaluating the efficiency of regenerative heat exchangers was noted in the works of Ramin et al. [15–18]. Some methods of reducing the dynamic component of the devices error for a more accurate measure of air temperature in regenerative heat exchangers are also considered there.

Conclusions

The use of the wave approach to modeling regenerative heat exchangers makes it possible to take into account the influence of the following factors:

• variable gas temperature at the inlet of the regenerative heat exchanger;

• processes of non-stationary heat conduction in packing elements;

• longitudinal thermal conductivity of the heat exchanger packing.

Despite appealing to complex mathematical apparatus to substantiate the wave method for calculating the heat regenerators, the procedure for calculating such a heat exchanger turns out to be relatively simple and convenient for computer calculations. This is explained that calculating the regenerative heat exchanger by the wave method requires repeated repetition of the same and relatively simple steps for each temperature wave.

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