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METHOD FOR CALCULATING THE DISSIPATION ENERGY DURING THE FLOW OF GENERALIZED-DISPLACED FLUID IN THE CHANNELS OF TECHNOLOGICAL EQUIPMENTEduard V. Biletsky¹, Elena V. Petrenko^{*2}, Dmitrij P. Semeniuk²¹ National Technical University «Kharkiv Polytechnic Institute», 2, Kyrpychova str., Kharkiv, 61002, Ukraine² State Biotechnological University, 44 Alchevskih str., Kharkiv, 61002, Ukraine

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Abstract

This paper considers the problem of determining the dissipation energy during the flow of a generalized-displaced fluid in the channels of technological equipment. It is known that during the flow of highly viscous non-Newtonian fluids, the problem of heating this substance arises. This is primarily due to the fact that during the transportation of the material, the dissipation mechanism takes place, which leads to overheating of the material. In its turn, this affects the changes in the physical and chemical properties of the material and the technical and economic indicators of the corresponding equipment. We propose a method for calculating the dissipation energy during the flow of a generalized-displaced fluid in the channels of screw machines. To solve this problem, we used the superposition method to construct fields of larger dimensions from fields of smaller dimensions with different boundary conditions. A channel of flat and rectangular shape is considered. Fluid movement is carried out in the longitudinal and longitudinal-transverse directions of the channel. To calculate the amount of energy dissipation of a generalized-displaced fluid, it is necessary to first divide the channel sections into sections with different expressions for the flow rate. At the same time, each of the subareas consists of two curvilinear triangles and one rectangle. The mandatory steps of the calculations are the breakdown of the rectangle of the cross section of the straight channel, and the calculation of the integrals from the derivatives of the velocity. The proposed method allows to calculate the energy of dissipative heat generation when calculating the optimal parameters of technological equipment.

Keywords: fluid; generalized-displaced; dissipation; flow; channel; calculation.

**МЕТОД ОБЧИСЛЕННЯ ЕНЕРГІЇ ДИСИПАЦІЇ ПРИ ТЕЧІЇ
УЗАГАЛЬНО-ЗРУШЕНОЇ РІДИНИ У КАНАЛАХ ТЕХНОЛОГІЧНОГО ОБЛАДНАННЯ**Едуард В. Білецький¹, Олена В. Петренко², Дмитро П. Семенюк³¹ Національний технічний університет «Харківський політехнічний інститут», Кирпичова, 2, Харків, 61002, Україна² Державний біотехнологічний університет, вул. Алчевських, 44, Харків 61002, Україна**Анотація**

У даній роботі розглянуто проблему визначення енергії дисипації під час течії узагально-зрушеної рідини у канал технологічного обладнання. Пропонується метод обчислення енергії дисипації під час течії узагально-зрушеної рідини в каналах шнекових машин. Для розв'язання даної задачі застосовано метод суперпозиції. Розглядається канал плоскої та прямокутної форми. Рух рідини здійснюється у поздовжньо та поздовжньо-поперечних напрямках каналу. Для обчислення величини енергії дисипації попередньо необхідно провести розбивку перетинів каналів на відповідні ділянки з різними виразами для швидкості течії. Розбивка прямокутника перетину прямого каналу та обчислення інтегралів від похідних швидкості є обов'язковими етапами розрахунків. Запропонований метод дозволяє обчислити енергію дисипативного тепловиділення в процесі розрахунку оптимальних параметрів технологічного обладнання.

Ключові слова: рідина; узагально-зрушена; дисипація; течія; канал; обчислення.

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Introduction

Thermal processes are the most common processes in chemical and food technologies [1–3]. Today, many scientific works describe the heat transfer of Newtonian fluids [4–5]. But there are quite a lot of chemical compounds (polymers, plastics, construction mixtures) and food products (chocolate, flour, confectionery) that belong to multicomponent systems that have a relatively high viscosity and a non-linear nature of the flow, these are the so-called non-Newtonian fluids [6–7].

To assess the structural and mechanical properties, it is necessary to know the type of structure, the dependence of stress on the displacement rate, as well as the amount of dissipation [8].

It is known that during its motion, a non-Newtonian fluid loses part of its energy, which turns into dissipative heat [9]. Knowledge of the amount of dissipation makes it possible to select technological equipment with the best power reserve, thereby reducing production costs [10].

Analysis of recent research and publications. The class of non-Newtonian fluids is very broad. It includes various media that have a certain degree of elasticity, their internal energy may depend on deformations; and the equation of state contains independent kinematic parameters [11].

An important example of non-Newtonian rheology are materials whose behavior is modeled by a non-Newtonian fluid, the viscosity of which depends on the shear rate, for example: polymers with fillers, gels, sols, macromolecular solutions [12].

From the analysis of the technical literature, it can be concluded that among the variety of non-Newtonian fluids, representatives of three classes are the most common: Bingham, generalized-displaced, and graded fluids [13]. By the term "generalized-displaced fluids" we mean fluids whose viscosity depends on the rate of

displacement in an arbitrary way [14]. In practice, taking into account the rheological features of the material is often reduced to an equation that relates viscosity to velocity gradients [15].

In the works of the authors [16–17] it is shown that when using various materials close in their physical properties to non-Newtonian fluids, they face a problem related to the dissipation of mechanical energy. High dynamic viscosity causes the dissipation of mechanical energy, which in turn leads to overheating of the material even at relatively low feed rates [18–20].

It is known that during the flow of non-Newtonian fluids with high viscosity in a straight channel, part of the energy is transformed into dissipative heat [21–22]. The paper [23] shows that the source of dissipative heat release is included in the heat exchange equation as a separate term. In [24–26], a method for determining the amount of dissipation during the flow of a viscoplastic fluid is proposed.

Results of the research and their discussion

In this work, we consider the flow of a generalized-displaced fluid. The movement of fluid is carried out in the longitudinal and longitudinal-transverse directions of the channel of flat and rectangular shape. To calculate the amount of energy dissipation of a generalized-displaced fluid, it is necessary to first divide the channel sections into corresponding sections with different expressions for the flow rate, this was discussed in detail in works [27–28].

The breakdown of flat and rectangular channels is presented in the figure, the breakdown elements are indicated by values S_y^\pm , S_x^\pm . To shorten the entries [29], different types of flow, which are correlated with their corresponding subareas of breakdowns, can be written in the form of equations, which will be given below.

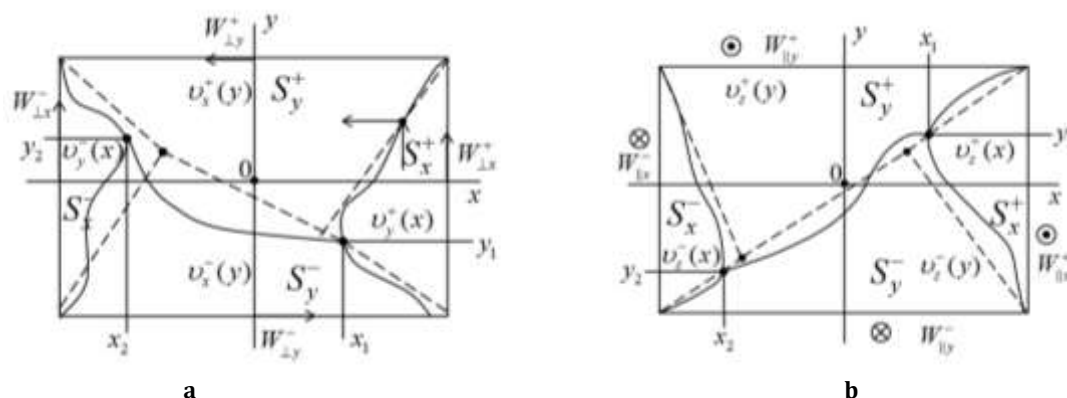


Fig. Split of the rectangular channel and linearization of the split
a - for cross-sectional flow; b - for longitudinal flow

The breakdown of the rectangle of the cross section of the straight channel and the calculation of the integrals from the derivatives of the velocity are mandatory stages of the calculations. For a viscoplastic fluid only the part of the cross section of the channel [24] that lies outside the solid core contributes to the dissipation, but the generalized-displaced fluid has fluidity over the entire cross section of the channel [10]. If cubic polynomials arise when performing integrations from the

$$\dot{E} = \iint ds^+ \left(\frac{\partial v^+}{\partial y} \right)_\mu^2 + \iint ds^- \left(\frac{\partial v^-}{\partial y} \right)_\mu^2;$$

$$\frac{\partial v^\pm}{\partial y} = \mp \frac{\alpha}{2\beta} + \left(\frac{\alpha^2}{4\beta^2} \pm \frac{y - y^*}{\beta} \frac{dP}{dz} \right)^{1/2},$$

where \dot{E} – dissipation on the intersection of flat channel, J/m³.

α and β – constants, the values of which are determined either experimentally or as a result of piecewise linear approximation of the graph of the

$$\dot{E} = \int_{-h}^{y^*} dy \left\{ \alpha \left(\frac{\partial v^-}{\partial y} \right)^2 + \beta \left| \frac{\partial v^-}{\partial y} \right| \left(\frac{\partial v^-}{\partial y} \right)^2 \right\} + \int_{y^*}^h dy \left\{ \alpha \left(\frac{\partial v^+}{\partial y} \right)^2 + \beta \left| \frac{\partial v^+}{\partial y} \right| \left(\frac{\partial v^+}{\partial y} \right)^2 \right\} \quad (3)$$

It follows from the last expression that it is necessary to be able to calculate integrals of this type:

$$\int \left(\frac{\partial v^\pm}{\partial y} \right)^m dy = \int dy \left\{ \pm \left[\frac{\alpha^2}{4\beta^2} \pm \frac{y - y^*}{\beta} \frac{dP}{dz} \right]^{1/2} \mp \frac{\alpha}{2\beta} \right\}^m \quad (4)$$

where $m = 2, 3$.

The base of the exponent must always be positive, because if $y^* \leq y \leq h$, then the sum of values in square brackets is greater than zero

$$\mp \frac{\alpha}{2\beta} \pm \left[\frac{\alpha^2}{4\beta^2} \pm \frac{y - y^*}{\beta} \frac{dP}{dz} \right]^{1/2} = \mp \frac{\alpha}{2\beta} + \omega^\pm; \quad (5)$$

$$\pm \frac{1}{\beta} \frac{dP}{dz} = 2\omega^\pm d\omega^\pm. \quad (6)$$

With the help of this change of variables, the integrals of degrees of derivatives can be numerically calculated. First, the integral from the

$$\begin{aligned} \int_{-h}^{y^*} d\omega^- \frac{2\beta}{dP/dz} \omega^- \left(\frac{\alpha}{2\beta} - \omega^- \right)^m &= \frac{2\beta}{dP/dz} \cdot \int_{-h}^{y^*} \left(\frac{\alpha}{2\beta} - \omega^- \right)^{m+1} d\omega^- - \frac{2\beta}{dP/dz} \int_{-h}^{y^*} \left(\frac{\alpha}{2\beta} - \omega^- \right)^m d\omega^- \times \frac{\alpha}{2\beta} = \\ &= \left\{ \frac{2\beta}{dP/dz} \left(\frac{\alpha}{2\beta} - \omega^- \right)^{m+2} \cdot \frac{2}{m+2} - \frac{2\beta}{dP/dz} \cdot \frac{\alpha}{2\beta} \left(\frac{\alpha}{2\beta} - \omega^- \right)^{m+1} \cdot \frac{1}{m+1} \right\} \Big|_{-h}^{y^*}. \end{aligned} \quad (7)$$

After substituting the limits of integration and performing some simple transformations, we get the following final result:

$$\begin{aligned} \int_{-h}^{y^*} dy \left(\frac{\partial v^-}{\partial y} \right)^m &= \frac{1}{m+1} \cdot \frac{\alpha}{dP/dz} \cdot \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h + y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^{m+2} - \\ &- \frac{1}{m+2} \cdot \frac{2\beta}{dP/dz} \cdot \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h + y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^{m+1}. \end{aligned} \quad (8)$$

derivatives of the flow velocity for a viscoplastic fluid, and the integration itself is performed without difficulty, then when integrating the derivatives of the velocity of a generalized-displaced fluid, certain transformations must be performed, which are given in this paper.

These transformations are listed in sequence below. In the case of flat longitudinal flow, the amount of dissipation applied to the cross section of the channel has the following form:

$$\mu = \alpha + \beta \left| \frac{\partial v^\pm}{\partial y} \right|, \quad (1)$$

$$\mu = \alpha + \beta \left| \frac{\partial v^\pm}{\partial y} \right|, \quad (2)$$

function $\mu = \mu(I_2)$, we considered this in detail in works [10; 27].

If we substitute the second expression into the first, taking into account the expression for viscosity μ , it turns out that the value \dot{E} takes the following form:

everywhere, except for $y = y^*$ where it equals zero. If $h \leq y \leq y^*$, then the same conditions apply. To calculate a power expression, we need to make the following change of variable:

lower branch in the interval from $-h$ to y^* is considered, while the following result has place:

A similar calculation of the integral of the velocity derivative in the interval from y^* to $+h$ leads to the following result:

$$\int_{y^*}^h d\omega^+ \frac{2\beta}{dP/dz} \omega^+ \left(\frac{\alpha}{2\beta} + \omega^+ \right)^m = \frac{2\beta}{dP/dz} \int_{y^*}^h \left(\frac{\alpha}{2\beta} + \omega^+ \right)^{m+1} d\omega^+ + \frac{2\beta}{dP/dz} \cdot \frac{\alpha}{2\beta} \int_{y^*}^h \left(\frac{\alpha}{2\beta} + \omega^+ \right)^m d\omega^+ = \left[\frac{2\beta}{dP/dz} \cdot \frac{1}{m+2} \cdot \left(\omega^+ - \frac{\alpha}{2\beta} \right)^{m+2} + \frac{2\beta}{dP/dz} \cdot \frac{\alpha}{2\beta} \cdot \frac{1}{m+1} \cdot \left(\omega^+ - \frac{\alpha}{2\beta} \right)^{m+1} \right]_{y^*}^h \quad (9)$$

After substituting the integration boundaries and performing simple transformations, we get the following final result:

$$\int_{y^*}^h dy \left(\frac{\partial v^+}{\partial y} \right)^m = \frac{1}{m+2} \cdot \frac{2\beta}{dP/dz} \left[\left(\frac{\alpha^2}{4\beta^2} + \frac{h-y^*}{\beta} \frac{dP}{dz} \right)^{1/2} - \frac{\alpha}{2\beta} \right]^{m+2} + \frac{1}{m+1} \cdot \frac{\alpha}{dP/dz} \cdot \left[\left(\frac{\alpha^2}{4\beta^2} + \frac{h-y^*}{\beta} \frac{dP}{dz} \right)^{1/2} - \frac{\alpha}{2\beta} \right]^{m+1} \quad (10)$$

Now we need to place (8) and (10) in (3), setting $m=2$ and $m=3$. As a result, for the corresponding integrals, we obtain:

$$\int_{-h}^{y^*} dy \left(\frac{\partial v^-}{\partial y} \right)^2 = \frac{1}{3} \cdot \frac{\alpha}{dP/dz} \cdot \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^3 - \frac{1}{4} \frac{2\beta}{dP/dz} \cdot \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^4; \quad (11)$$

$$\int_{y^*}^h dy \left(\frac{\partial v^-}{\partial y} \right)^2 = \frac{1}{3} \cdot \frac{\alpha}{dP/dz} \cdot \left[\left(\frac{\alpha^2}{4\beta^2} + \frac{h-y^*}{\beta} \frac{dP}{dz} \right)^{1/2} - \frac{\alpha}{2\beta} \right]^3 + \frac{1}{4} \frac{2\beta}{dP/dz} \cdot \left[\left(\frac{\alpha^2}{4\beta^2} + \frac{h-y^*}{\beta} \frac{dP}{dz} \right)^{1/2} - \frac{\alpha}{2\beta} \right]^4; \quad (12)$$

$$\int_{-h}^{y^*} dy \left(\frac{\partial v^-}{\partial y} \right)^3 = \frac{1}{4} \cdot \frac{\alpha}{dP/dz} \cdot \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^4 - \frac{1}{5} \frac{2\beta}{dP/dz} \times \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^5; \quad (13)$$

$$\int_{y^*}^h dy \left(\frac{\partial v^+}{\partial y} \right)^3 = \frac{1}{4} \cdot \frac{\alpha}{dP/dz} \cdot \left[-\frac{\alpha}{2\beta} + \left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^4 + \frac{1}{5} \frac{2\beta}{dP/dz} \times \left[\left(\frac{\alpha^2}{4\beta^2} + \frac{h+y^*}{\beta} \frac{dP}{dz} \right)^{1/2} - \frac{\alpha}{2\beta} \right]^5 \quad (14)$$

Substitution (11)...(14) in (3) gives an expression for determining the energy dissipation of the generalized-displaced flow. The compact notation of this expression is the following power form:

$$\dot{E} = \frac{1}{3} \frac{\alpha^2}{dP/dz} (W_{\square y}^{+3} + W_{\square y}^{-3}) + \frac{3}{4} \frac{\alpha\beta}{dP/dz} (W_{\square y}^{+4} + W_{\square y}^{-4}) - \frac{2}{5} \frac{\beta^2}{dP/dz} (W_{\square y}^{-5} - W_{\square y}^{+5}), \quad (15)$$

$$w^\pm = \mp \frac{\alpha}{2\beta} \pm \left(\frac{\alpha^2}{4\beta^2} + \frac{h \mp y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \quad (16)$$

Next, the case of flat longitudinal-transverse flow is considered. When calculating the dissipation energy of this type of flow, it is necessary to calculate the following integral:

$$\dot{E} = \int_{-h}^{+h} dy \left\{ \alpha \left[\left(\frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} \right)^2 \right] + \beta \left[\left(\frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} \right)^2 \right]^{3/2} \right\} \quad (17)$$

The first integral of the terms with the factor α is reduced to the sum of the integrals that were calculated for a flat longitudinal flow.

The second integral is impossible to calculate in the obvious way. In addition, different derivatives are calculated in different intervals due to the fact that the values y_z^* and y_x^* are not equal to each other.

If we consider the integral with factor β it should be taken into account that the expressions for the velocities $v_z(y)$ and $v_x(y)$ actually depend

$$\int_{-h}^{+h} dy \left[\left(\frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} \right)^2 \right]^{3/2} = \int_{-h}^{y_z^*} dy \left[\left(\frac{\partial v_z^-}{\partial y} \right)^2 + \left(\frac{\partial v_x^-}{\partial y} \right)^2 \right]^{3/2} + \quad (18)$$

$$+ \int_{y_z^*}^{y_x^*} dy \left[\left(\frac{\partial v_z^-}{\partial y} \right)^2 + \left(\frac{\partial v_x^+}{\partial y} \right)^2 \right]^{3/2} + \int_{y_x^*}^{+h} dy \left[\left(\frac{\partial v_z^+}{\partial y} \right)^2 + \left(\frac{\partial v_x^+}{\partial y} \right)^2 \right]^{3/2}.$$

$$\int_{y_x^*}^{+h} dy \left[\left(\frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} \right)^2 \right]^{3/2} = \int_{-h}^{y_z^*} dy \left[\left(\frac{\partial v_z^-}{\partial y} \right)^2 + \left(\frac{\partial v_x^-}{\partial y} \right)^2 \right]^{3/2} + \quad (19)$$

$$+ \int_{y_z^*}^{y_x^*} dy \left[\left(\frac{\partial v_z^+}{\partial y} \right)^2 + \left(\frac{\partial v_x^-}{\partial y} \right)^2 \right]^{3/2} + \int_{y_x^*}^{+h} dy \left[\left(\frac{\partial v_z^+}{\partial y} \right)^2 + \left(\frac{\partial v_x^+}{\partial y} \right)^2 \right]^{3/2}.$$

Next, in the same way as it is done for a purely longitudinal plane flow, we should enter the variables $W_{\perp y}^{\pm}$ and $W_{\perp x}^{\pm}$ according to the following rules:

$$W_{\perp y}^{\pm} = \left(\frac{\alpha_z^2}{4\beta_z^2} \pm \frac{y - y_z^*}{\beta_z} \frac{\partial P}{\partial z} \right)^{1/2}; \quad W_{\perp x}^{\pm} = \left(\frac{\alpha_x^2}{4\beta_x^2} \pm \frac{y - y_x^*}{\beta_x} \frac{\partial P}{\partial x} \right)^{1/2}. \quad (20)$$

In both cases (18) and (19), it is necessary to be able to calculate integrals of this type in the specified intervals:

$$\int dy \left\{ \left[\mp \frac{\alpha_z}{2\beta_z} + W_{\perp y}^{\pm} \right]^2 + \left[\mp \frac{\alpha_x}{2\beta_x} + W_{\perp x}^{\pm} \right]^2 \right\}^{3/2}. \quad (21)$$

Neglecting accuracy, the integrand can be represented as a linear combination of powers of expressions in square brackets. At the same time, based on the results of calculations for a flat longitudinal flow, it is possible to note that cubic expressions must necessarily be present in this linear combination. Further, bearing in mind that

$$\left[\left(\frac{\partial v_z^{\pm}}{\partial y} \right)^2 + \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^2 \right]^{3/2} \approx c_1 \left(\frac{\partial v_z^{\pm}}{\partial y} \right)^3 + c_2 \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^3 + c_3 \left(\frac{\partial v_z^{\pm}}{\partial y} \right)^2 + c_4 \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^2, \quad (22)$$

$$\left[\left(\frac{\partial v_z^{\pm}}{\partial y} \right)^2 + \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^2 \right]^{3/2} \approx c'_1 \left(\frac{\partial v_z^{\pm}}{\partial y} \right)^3 + c'_2 \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^3 + c'_3 \left(\frac{\partial v_z^{\pm}}{\partial y} \right)^2 + c'_4 \left(\frac{\partial v_x^{\pm}}{\partial y} \right)^2. \quad (23)$$

where constants c_i and c'_i , $i=1 \div 4$ somewhat different from each other.

A more accurate and correct representation for the left part of (22) and (23) should include six

not on α and β , but on α_z , β_z and α_x , β_x correspondingly. Expression of these values through α i β is shown in the work [10].

To perform the integration of the second integral, we should first determine which of the values y_z^* and y_x^* is bigger. Their specific ratio is not important for further construction; therefore, it is further assumed that $y_x^* < y_z^*$. In a general consideration, the following representations hold for the indicated inequality, as well as for the opposite one:

the coefficients of the linear combination are easy to determine using the value of the complete integrand expression (21) at the points $y = \pm h$, $y = y_z^*$, $y = y_x^*$, the integrand expression is presented in the form:

constants and in addition to the cubic, should contain second- and first-degree derivatives of the velocities. But in this case, the determination of these constants requires a system of linear equations of the sixth order, for which specific

expressions for the constants c_i ($i=1\div 6$) are extremely cumbersome. Bearing in mind that the general case can always be solved numerically, the case with four constants is considered next. For concreteness, the case of expansion of expression (22) is chosen, while it should be noted that case (23) is no different from it. To increase the accuracy, it is possible to separately solve the systems of equations (22) and (23) twice for c_i

$$\left\{ \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(h) \right]^2 + \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(h) \right]^2 \right\}^{3/2} = c_1 \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(h) \right]^3 + c_2 \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(h) \right]^3 + c_3 \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(h) \right]^2 + c_4 \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(h) \right]^2. \quad (24)$$

$$\left\{ \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(-h) \right]^2 + \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(-h) \right]^2 \right\}^{3/2} = c_1 \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(-h) \right]^3 + c_2 \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(-h) \right]^3 + c_3 \left[-\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^+(-h) \right]^2 + c_4 \left[-\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^\pm(-h) \right]^2. \quad (25)$$

The following equations are obtained at points $y = y_z^*$ and $y = y_x^*$:

$$\left[\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^+(y_z^*) \right]^3 = c_2 \left[\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^+(y_z^*) \right]^2 + c_4 \left[\frac{\alpha_x}{2\beta_x} + W_{\text{lx}}^+(y_z^*) \right]^2. \quad (26)$$

$$\left[\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^-(y_x^*) \right]^3 = c_1 \left[\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^-(y_x^*) \right]^2 + c_4 \left[\frac{\alpha_z}{2\beta_z} + W_{\text{ly}}^-(y_x^*) \right]^2. \quad (27)$$

Any pair of constants can easily be excluded from the recent equations so that the entire system of equations is reduced to a system of

$$\begin{aligned} [h]_z &\equiv \tilde{a}; & [-h] &\equiv \tilde{d}; \\ [-h]_z &\equiv \tilde{b}; & [y_x^*] &\equiv \tilde{e}; \\ [h]_x &\equiv \tilde{c}; & [y_z^*] &\equiv \tilde{f}. \end{aligned} \quad (28)$$

In formulas (24)–(27), the notation of the corresponding values is given in square brackets.

It should be emphasized that these notations are effective only when solving the system of

$$\begin{aligned} (\tilde{a}^2 + \tilde{c}^2)^{3/2} &= \tilde{c}_1 \tilde{a}^3 + \tilde{c}_2 \tilde{c}^3 + \tilde{c}_3 \tilde{a}^2 + \tilde{c}_4 \tilde{c}^2; \\ (\tilde{b}^2 + \tilde{d}^2)^{3/2} &= \tilde{c}_1 \tilde{b}^3 + \tilde{c}_2 \tilde{d}^3 + \tilde{c}_3 \tilde{b}^2 + \tilde{c}_4 \tilde{d}^2; \\ \tilde{f}^3 &= 0 + \tilde{c}_2 \tilde{f}^3 + 0 + \tilde{c}_4 \tilde{f}^2; \\ \tilde{e}^3 &= \tilde{c}_1 \tilde{e}^3 + 0 + \tilde{c}_3 \tilde{e}^2 + 0. \end{aligned} \quad (29)$$

Except for constants \tilde{c}_3 and \tilde{c}_4 for \tilde{c}_1 and \tilde{c}_2 we get the following system of equations:

$$\begin{aligned} (\tilde{a}^2 + \tilde{c}^2)^{3/2} - \tilde{a}^2 \tilde{e} - \tilde{c}^2 \tilde{f} &= \tilde{c}_1 \tilde{a}^2 (\tilde{a} - \tilde{c}) + \tilde{c}_2 \tilde{c}^2 (\tilde{c} - \tilde{f}) \\ (\tilde{b}^2 + \tilde{d}^2)^{3/2} - \tilde{b}^2 \tilde{e} - \tilde{d}^2 \tilde{f} &= \tilde{c}_1 \tilde{b}^2 (\tilde{b} - \tilde{e}) + \tilde{c}_2 \tilde{d}^2 (\tilde{d} - \tilde{f}). \end{aligned} \quad (30)$$

If we omit intermediate cumbersome calculations for \tilde{c}_1 and \tilde{c}_2 , the following solutions hold:

and c_i' , and then take their average arithmetic sum $(c_i + c_i')/2$.

Representations (22) and (23) reduce the problem of integration (18) and (19) to an already solved problem about a flat longitudinal flow.

Finding the coefficients $c_1 \dots c_4 c_1' \dots c_4'$ leads to the following equations at the points $y = \pm h$:

equations of the second order. To shorten the record, it is useful to enter the following abbreviations:

equations (24)–(27). The system of equations can be rewritten using (28) in the following short form:

$$\tilde{c}_1 = \frac{\left[(\tilde{a}^2 + \tilde{c}^2)^{3/2} - \tilde{a}^2 \tilde{e} - \tilde{c}^2 \tilde{f} \right] \tilde{d}^2 (\tilde{d} - \tilde{f}) - \left[(\tilde{b}^2 + \tilde{d}^2)^{3/2} - \tilde{b}^2 \tilde{e} - \tilde{d}^2 \tilde{f} \right] \tilde{c}^2 (\tilde{c} - \tilde{f})}{\tilde{a}^2 \tilde{d}^2 (\tilde{a} - \tilde{e})(\tilde{d} - \tilde{f}) - \tilde{b}^2 \tilde{c}^2 (\tilde{b} - \tilde{e})(\tilde{c} - \tilde{f})}; \quad (31)$$

$$\tilde{c}_2 = \frac{\left[(\tilde{b}^2 + \tilde{d}^2)^{3/2} - \tilde{b}^2 \tilde{e} - \tilde{d}^2 \tilde{f} \right] \tilde{a}^2 (\tilde{a} - \tilde{e}) - \left[(\tilde{a}^2 + \tilde{c}^2)^{3/2} - \tilde{a}^2 \tilde{e} - \tilde{c}^2 \tilde{f} \right] \tilde{b}^2 (\tilde{b} - \tilde{e})}{\tilde{a}^2 \tilde{d}^2 (\tilde{a} - \tilde{e})(\tilde{d} - \tilde{f}) - \tilde{b}^2 \tilde{c}^2 (\tilde{b} - \tilde{e})(\tilde{c} - \tilde{f})}. \quad (32)$$

Constants \tilde{c}_3 and \tilde{c}_4 can be found from the relations of the following form:

$$\tilde{c}_3 = \tilde{e}(1 - \tilde{c}_1), \quad \tilde{c}_4 = \tilde{f}(1 - \tilde{c}_2). \quad (33)$$

Next, it is necessary to determine the values $\tilde{c}'_1 \dots \tilde{c}'_4$ for expanding (24) and (25). The given system of equations is similar to (29) and takes the following form in notation (28):

$$\begin{aligned} (\tilde{a}^2 + \tilde{c}^2)^{3/2} &= \tilde{c}'_1 \tilde{a}^3 + \tilde{c}'_2 \tilde{c}^3 + \tilde{c}'_3 \tilde{a} + \tilde{c}'_4 \tilde{c}; \\ (\tilde{b}^2 + \tilde{d}^2)^{3/2} &= \tilde{c}'_1 \tilde{b}^3 + \tilde{c}'_2 \tilde{d}^3 + \tilde{c}'_3 \tilde{b} + \tilde{c}'_4 \tilde{d}; \\ \tilde{f}^3 &= 0 + \tilde{c}'_2 \tilde{f}^3 + 0 + \tilde{c}'_4 \tilde{f}; \\ \tilde{e}^3 &= \tilde{c}'_1 \tilde{e}^3 + 0 + \tilde{c}'_3 \tilde{e} + 0. \end{aligned} \quad (34)$$

Next, it is necessary to carry out the same procedure as when solving the system of equations (29), that is, exclude \tilde{c}'_3 and \tilde{c}'_4 . Omitting all intermediate calculations, the finished result looks like this:

$$\tilde{c}'_1 = \frac{\left[(\tilde{a}^2 + \tilde{c}^2)^{3/2} - \tilde{a} \tilde{e}^2 - \tilde{c} \tilde{f}^2 \right] \tilde{d} (\tilde{d}^2 - \tilde{f}^2) - \left[(\tilde{b}^2 + \tilde{d}^2)^{3/2} - \tilde{b} \tilde{e}^2 - \tilde{d} \tilde{f}^2 \right] \tilde{c} (\tilde{c}^2 - \tilde{f}^2)}{\tilde{a} \tilde{d} (\tilde{a}^2 - \tilde{e}^2)(\tilde{d}^2 - \tilde{f}^2) - \tilde{b} \tilde{c} (\tilde{b}^2 - \tilde{e}^2)(\tilde{c}^2 - \tilde{f}^2)}; \quad (35)$$

$$\tilde{c}'_2 = \frac{\left[(\tilde{b}^2 + \tilde{d}^2)^{3/2} - \tilde{b} \tilde{e}^2 - \tilde{d} \tilde{f}^2 \right] \tilde{a} (\tilde{a}^2 - \tilde{e}^2) - \left[(\tilde{a}^2 + \tilde{c}^2)^{3/2} - \tilde{a} \tilde{e}^2 - \tilde{c} \tilde{f}^2 \right] \tilde{b} (\tilde{b}^2 - \tilde{e}^2)}{\tilde{a} \tilde{d} (\tilde{a}^2 - \tilde{e}^2)(\tilde{d}^2 - \tilde{f}^2) - \tilde{b} \tilde{c} (\tilde{b}^2 - \tilde{e}^2)(\tilde{c}^2 - \tilde{f}^2)}; \quad (36)$$

$$\tilde{c}'_3 = \tilde{e}^2 (1 - \tilde{c}_1); \quad \tilde{c}'_4 = \tilde{f}^2 (1 - \tilde{c}_2). \quad (37)$$

The values \tilde{a} , \tilde{c} , \tilde{e} , \tilde{b} , \tilde{d} , \tilde{f} in (31), (32), (33), (35), (36) have expressions through the derivatives of the velocity components according to the following rules:

$$\begin{aligned} \tilde{a} &= \frac{\partial v_z^+}{\partial y}(h); & \tilde{c} &= \frac{\partial v_x^+}{\partial y}(h); & \tilde{e} &= \frac{\partial v_z^-}{\partial y}(y_x^*); & \text{condition } y_x^* \leq y_z^*, & (38) \\ \tilde{b} &= \frac{\partial v_z^-}{\partial y}(-h); & \tilde{d} &= \frac{\partial v_x^-}{\partial y}(-h); & \tilde{f} &= \frac{\partial v_x^+}{\partial y}(y_z^*) \end{aligned}$$

If the opposite inequality is fulfilled, then the following should be understood by \tilde{e} and \tilde{f} :

$$\tilde{e} = \frac{\partial v_z^+}{\partial y}(y_x^*); \quad \tilde{f} = \frac{\partial v_x^-}{\partial y}(y_z^*); \quad \text{condition } y_z^* \leq y_x^*. \quad (39)$$

where \tilde{a} , \tilde{c} , \tilde{b} , \tilde{d} - have the same form as in (38).

In the future, taking into account the equations obtained above, it should be taken into account that the algorithm for calculating the dissipation energy for a flat longitudinal-transverse flow is based on the use of the algorithm for calculating the dissipation energy for a flat longitudinal flow.

Next, the longitudinal-transverse flow in a rectangular channel is considered. Let's turn to the diagram of the breakdown of the cross section of the channel for the longitudinal-transverse flow

(see fig.), which has already been discussed in detail in works [10, 24, 29].

As can be seen from the figure, the longitudinal and transverse currents are constructed in such a way that in each element of the rectangular partition there is a longitudinal current with a derivative component, which includes a set of components $\partial v_{zi} / \partial x_k$, where $i, k = x, y$ and a transverse current with a derivative component with a set of components $\partial v_i^\pm / \partial x_k$, where

$i, k = x, y$. That is, all elements of the breakdown can be divided into two subsets. In one subset, each partition element corresponds to a pair of velocity derivatives along one coordinate, and in

the other subset, each partition element corresponds to a pair of derivatives along different coordinates.

Given the above, the following pairs of derivatives arise for the first subset:

$$\frac{\partial v_z^+}{\partial y} \text{ and } \frac{\partial v_x^+}{\partial y}; \quad \frac{\partial v_z^-}{\partial y} \text{ and } \frac{\partial v_x^-}{\partial y}; \quad \frac{\partial v_z^+}{\partial x} \text{ and } \frac{\partial v_y^+}{\partial x}; \quad \frac{\partial v_z^-}{\partial x} \text{ and } \frac{\partial v_y^-}{\partial x}.$$

For the second subset, pairs of derivatives will include derivatives in different coordinates in the following pairs:

$$\frac{\partial v_z^+}{\partial y} \text{ and } \frac{\partial v_y^+}{\partial x}; \quad \frac{\partial v_z^-}{\partial y} \text{ and } \frac{\partial v_y^-}{\partial x} \text{ and so on.}$$

For all these combinations of derivatives, it is necessary to calculate integrals of the following form:

$$\iint dy dx \left\{ \left[\alpha + \beta \sqrt{\left(\frac{\partial v_i^\pm}{\partial x_k} \right)^2 + \left(\frac{\partial v_j^\pm}{\partial x_k} \right)^2} \right] \cdot \left[\left(\frac{\partial v_i^\pm}{\partial x_k} \right)^2 + \left(\frac{\partial v_j^\pm}{\partial x_k} \right)^2 \right] \right\}; \quad (40)$$

$$\iint dy dx \left\{ \left[\alpha + \beta \sqrt{\left(\frac{\partial v_i^\pm}{\partial x_k} \right)^2 + \left(\frac{\partial v_j^\pm}{\partial x_l} \right)^2} \right] \cdot \left[\left(\frac{\partial v_i^\pm}{\partial x_k} \right)^2 + \left(\frac{\partial v_j^\pm}{\partial x_l} \right)^2 \right] \right\}. \quad k \neq l \quad (41)$$

For the integral (40), the calculation can be carried out in the same way as for the plane longitudinal-transverse flow, that is, by presenting the fractional degree of the sum of squares of the derivatives through the sum of whole degrees. In this case, the following values $y = \pm h, y_z^*, x = \pm a, y_z^*$, $y = \pm h, y_x^*, x = \pm a, x_y^*$ should be selected as location points.

Unlike the cases of longitudinal and longitudinal-transverse flow in the case under consideration, the integration is carried out along

$$\int_{x_0}^{-a} dx \int_{-h}^{y^-(x)} \left(\frac{\partial v_z^-}{\partial y} \right)^m dy, \quad (42)$$

where $m = 2, 3$.

The result of the first integration looks like this:

$$\int_{-h}^{y^-(x)} dy \left(\frac{\partial v_z^-}{\partial y} \right)^m = \left\{ \frac{2\beta}{dP/dz} \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} - \frac{y^-(x) - y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^{m+2} \right\} \times$$

$$\times \frac{1}{m+2} - \frac{\alpha}{dP/dz} \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} - \frac{y^-(x) - y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right] -$$

$$\left\{ \frac{2\beta}{dP/dz} \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h + y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^{m+2} \right\} \frac{1}{m+2} - \frac{\alpha}{dP/dz} \left[\frac{\alpha}{2\beta} - \left(\frac{\alpha^2}{4\beta^2} + \frac{h + y^*}{\beta} \frac{dP}{dz} \right)^{1/2} \right]^{m+1} \cdot \frac{1}{m+1} \right\}. \quad (43)$$

where $y^-(x)$ – given function from x .

If we use the substitution (20), then the terms of the last expression, which includes the function $y^-(x)$ acquire a simple form for integration along the x coordinate only if $y^-(x)$ is a linear function. If the lines of the boundaries of the breakdown

one of the coordinates to the border of the corresponding contour of the rectangle of the channel cross section into elements, and along the other coordinate within the specified limits. These limits are determined by the coordinates of the point of intersection of the contours of the breakdown limits and values $\pm a, \pm h$ (see figure).

As an example, below we consider the calculation of the integral for the degree of the derivative $\partial v_z^- / \partial y$ of the following form:

strongly deviate from straight lines, then the corresponding integrals can only be calculated numerically.

Conclusions

The proposed method allows to calculate the dissipation energy in the longitudinal and longitudinal-transverse directions of the flow of

the generalized-displaced fluid in the flat and rectangular channels of the technological equipment. The obtained equations make it possible to determine the temperatures of the working material in the channel and the coolant along the length of the flat and rectangular channels for various cases of heat exchange, the heat transfer coefficients of the flow of a generalized displaced fluid with an arbitrary distribution of velocity at the channel boundaries.

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