Journal of Chemistry and Technologies, 2025, *33*(2), 538-547

Journal of Chemistry and Technologies

pISSN 2663-2934 (Print), ISSN 2663-2942 (Online).

journal homepage: http://chemistry.dnu.dp.ua
editorial e-mail: chem.dnu@gmail.com

UDC 532.542:532.58:519.6

MODELING OF THE FLOW OF NON-NEWTONIAN FLUIDS USING THE SUPERPOSITION METHOD

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Received 17 January 2025; accepted 18 May 2025; available online 15 July 2025

Abstract

This paper addresses the challenges of simulating viscoplastic longitudinal and cross-sectional flows of non-Newtonian fluids. A superposition method is proposed to construct higher-dimensional flow fields from lower-dimensional ones, accommodating varying boundary conditions and pressure-dependent rheological parameters. The study details theoretical approaches for modeling non-Newtonian fluid flows in channels with diverse geometries, including moving boundaries and pressure drops at channel edges, considering the functional relationships between key process parameters. It is demonstrated that both longitudinal and cross-sectional flows can be represented as a combination of one-dimensional longitudinal flows of the same type, enabling the description of three-dimensional isothermal flows in rectangular channels and two-dimensional flows in flat channels with varying aspect ratios. The resulting theoretical two- and three-dimensional models of viscous flows in basic channel geometries facilitates the investigation of fundamental process regularities and the determination of optimal macro-kinetic and macro-dynamic flow characteristics for non-Newtonian materials, ultimately aiming to reduce energy consumption and material usage in food processing equipment.

Keywords: flow; non-Newtonian fluid; channel; rheology; simulation.

МОДЕЛЮВАННЯ ТЕЧІЇ НЕНЬЮТОНОВСЬКИХ РІДИН МЕТОДОМ СУПЕРПОЗИЦІЇ

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Анотація

У роботі досліджені проблеми моделювання в'язкопластичних течій неньютонівських рідин у поздовжньому та поперечному напрямках. Запропоновано метод суперпозиції для побудови полів течії вищої розмірності на основі полів нижчої розмірності з урахуванням змінних граничних умов та залежності реологічних параметрів від тиску. Розглянуто теоретичні підходи до моделювання течій неньютонівських рідин у каналах різної конфігурації, зокрема з рухомими межами та перепадами тиску на краях, з урахуванням функціональних зв'язків між ключовими параметрами процесу. Показано, що поздовжні та поперечні течії можуть бути зведені до комбінації одновимірних поздовжніх течій аналогічного типу, що дозволяє описувати тривимірні ізотермічні течії у прямокутних каналах та двовимірні течії у плоских каналах з різним співвідношенням сторін. Побудовано теоретичні дво- та тривимірні моделі в'язкопластичних течій у каналах базової геометрії, що дає змогу досліджувати основні закономірності процесу та визначати оптимальні макрокінетичні та макродинамічні характеристики течії неньютонівських матеріалів з метою оптимізації енергоспоживання та використання матеріалів у харчовому обладнанні.

Ключові слова: течія; неньютонівська рідина; канал; реологія; моделювання.

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doi: 10.15421/jchemtech.v33i2.321011

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Introduction

Understanding fluid hydrodynamics is fundamental to optimizing most food technology processes. Moving fluids transfer energy, including kinetic and thermal energy [1; 2]. Many fluids encountered in food processing equipment are heterogeneous systems, often consisting of solutions and mixtures exhibiting non-Newtonian behavior [3; 4]. Characterizing the structure and flow regimes of these non-Newtonian materials is crucial for designing efficient technological processes and optimizing hydrodynamic, thermal, and mass transfer indicators. However, the complexity of rheological models often limits the ability to fully describe the relationships between process parameters. This limitation hinders the development of new energy-efficient technologies and can lead to increased development time and costs [5; 6].

Therefore, developing scientifically sound approaches for modeling the flow of non-Newtonian materials remains an important challenge. This involves constructing theoretically robust mathematical models to describe non-Newtonian flow and identifying optimal structural and technological parameters for energy-efficient processes and equipment in the chemical and food industries [7].

Fluid flows within equipment can be broadly classified based on the Reynolds number (Re): Re > 1 (high Reynolds number) and Re < 1 (low Reynolds number). High Re flows, typical of low-viscosity liquids, facilitate efficient heat and mass transfer [8–10]. Conversely, low Re flows, characteristic of highly viscous liquids, generate high shear rates, internal friction, and pressure. These conditions can significantly alter the internal structure of the fluid. While heat and mass transfer are paramount in high Re flows, with the rheological state of the fluid playing a less significant role, the rheological behavior is of primary importance in low Re flows. Consequently, using only Newtonian models to describe these flows is inadequate.

A key distinction between high and low Re flows is the behavior of rheological parameters. In high Re flows, these parameters typically remain constant during the flow. In contrast, they can vary significantly in low Re flows, particularly in chemical processing. The driving force in high Re flows is typically a pressure difference across the flow domain. In low Re flows, the movement of the flow boundaries is the primary driving force. Transporting highly viscous liquids requires more energy, and the resulting pressure is a consequence of this boundary movement [11–14].

Examples of high Re flows include flow through tubes with a defined pressure drop, commonly found in various technological equipment and conduits [15–17]. Low Re flows are often observed in channels and within the working chambers of screw machines, where highly viscous fluids are processed and their properties modified [18–20]. While some general fluid mechanics principles apply to both Newtonian and non-Newtonian fluids, the distinct characteristics of low Re flows necessitate careful consideration of the fluid's rheological properties.

The aim of this work is to develop and improve theoretical models of viscoplastic flows of non-Newtonian fluids in channels of various configurations, taking into account complex interactions such as moving boundary conditions, variable rheological parameters depending on pressure, and three-dimensional effects. The research is aimed at optimizing the macrokinetic and macrodynamic characteristics of these flows, in particular to reduce energy consumption and more efficient use of materials in technological equipment.

Results and discussion.

In this paper, the methods for simulating viscoplastic longitudinal flow in flat and rectangular channels are discussed. The channel bounds are movable. This movement can occur both along and across the channel. The channel with rectangular cross-section is considered standard. The flow in the channel is characterized by velocity and pressure values in each point of the flow region. Information about the flow may be condensed (pressure and consumption only) and full, or local (pressure and velocity) at each point. Movement of liquid in the channel can be straight and curved. The latter does not affect the results because inertia does not matter for the flows with Reynolds number lower than 1 [21–23].

The equations for stokes flows have the following general form:

$$\begin{gathered} -\nabla P+\nabla\hat{\tau}=0\,,\ \vec{\upsilon}=\left(\upsilon_x,\upsilon_y,\upsilon_z\right), \\ \nabla\rho\vec{\upsilon}=0\,,\ \hat{\tau}=\tau_{ik}\,,\ i,k=x,y,z\,, \\ \hat{\tau}=\hat{\tau}\left(\hat{\dot{\varepsilon}},P,T\right),\ \hat{\dot{\varepsilon}}=\dot{\varepsilon}_{ik}\,,\ \rho=\rho(P), \end{gathered} \tag{1}$$

where P – pressure in the fluid, Pa;

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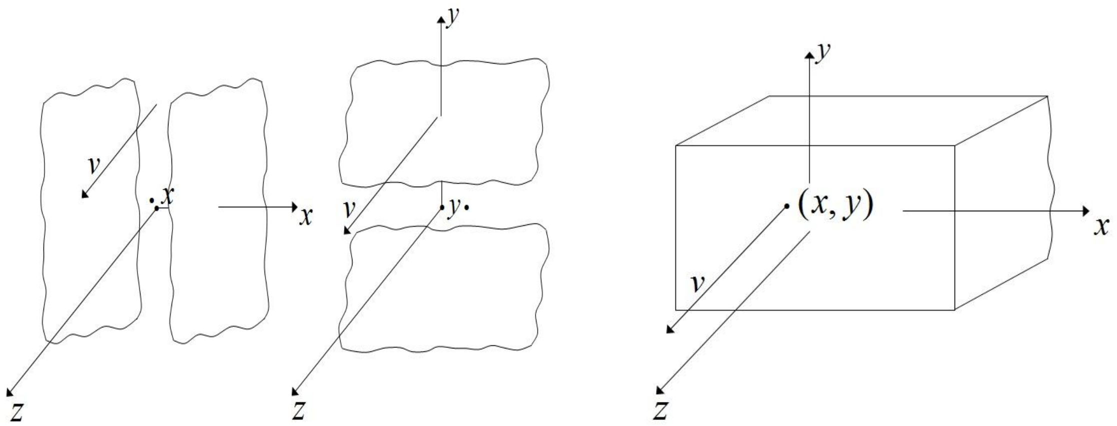
ρ – density of the fluid, kg/m^3;
$\hat{\tau}$ – stress tensor, Pa;
$\bar{\upsilon}$ – flow velocity vector, m/s;
T – temperature, K;
$\hat{\dot{\varepsilon}}$ – strain rate tensor, 1/s;
x, y, z – coordinates of point in the flow region, m.

All flows described by equations (1) can be divided into two groups. The first group includes flows with velocity vector which has only one component. This component may depend on one or two coordinates, but these coordinates should be transverse. If coordinate z is chosen as the longitudinal coordinate (along $0z$ axis), then coordinates x and y will be transverse. Longitudinal flows have a velocity component υ_z which can depend either on x or on y separately or on both these coordinates. Longitudinal flows have only one velocity component which depends only on transverse coordinates and can not contain any values which depend on pressure and temperature except for the pressure gradient. In these longitudinal flows the distribution of velocity is the same in all cross-sections. The second group contains flows with a velocity vector which has two or three components each of which depends on two or three coordinates.

The flows of the second group can be ordered like this: two-dimensional longitudinal flows; two-dimensional transverse flows with zero longitudinal velocity, three-dimensional twist-and-steer flows which contain all three velocity vector components each of which depends on all three coordinates.

The reasons for various types of flows are the boundary conditions and dependency (or independency) of rheological characteristics on pressure and temperature. Such dependency of reasons and consequences can be illustrated by the example of Newtonian or non-classical non-Newtonian fluid with properties that do not depend on the strain rate tensor. If some flow depends only on longitudinal coordinate and has a longitudinal component only than this is the flow in a flat channel which has only one pair of bounds – and the velocities of these bounds are also longitudinal. At the same time, only pressure varies along the channel. If rheological parameters depend on pressure, than component of the stress tensor in equation (1) will also depend on pressure. In this case, longitudinal velocity depends on longitudinal coordinate. Due to the equation of matter conservation (1) the second – transverse – velocity component appears, although boundary conditions are purely longitudinal. For Newtonian fluid, the longitudinal flows with one velocity component which depends on two transverse coordinates are possible. These flows demand for one additional pair of bounds with longitudinal boundary conditions to be available. Complication of this problem and the addition of dependency for rheological characteristics leads to the solution of this problem beyond the two-component flow. The flow obtains an additional component and yet another coordinate as an argument. Thus adding another pair of bounds adds new coordinate while adding dependency from the pressure adds both component and coordinate. This is illustrated on Fig. 1.



(a) (b)

Fig. 1. Longitudinal fluid flow:
(a) – in channels with mutually perpendicular bounds; (b) – in the rectangular channel

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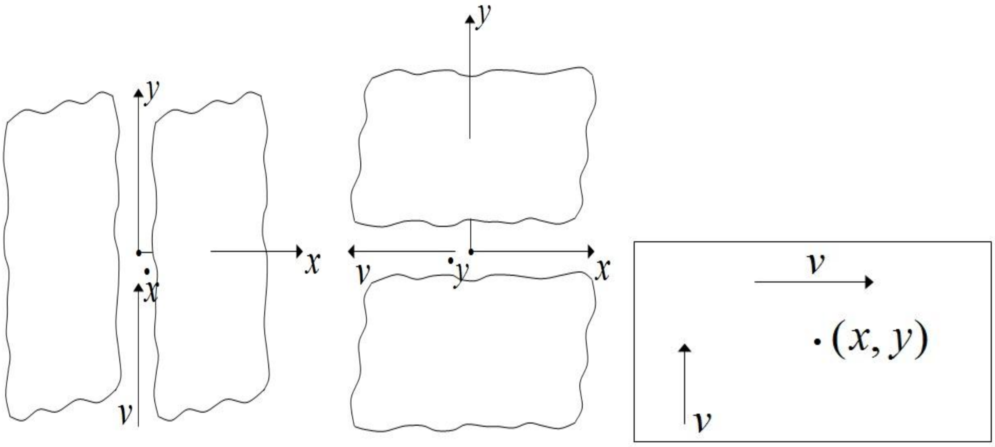
There are also purely transverse flows for Newtonian fluids. If the channel is flat and the velocities of its bounds are purely transverse, then the flow will be purely transverse and will depend on one transverse coordinate only. In the practical aspect, these flows are of interest to the small channels with large width. These channels may be approximately considered as flat. The flow consumption in wide closed channel has a value of zero. Hence, in order for the transverse flow in a flat channel to adequately represent the transverse flow in a rectangular channel, a purely longitudinal flow with zero consumption should be considered. If rheological characteristics of the flow depend on pressure, then the transverse flow in a flat channel obtains an additional velocity component and additional coordinate as a variable. This is illustrated on Fig. 2.

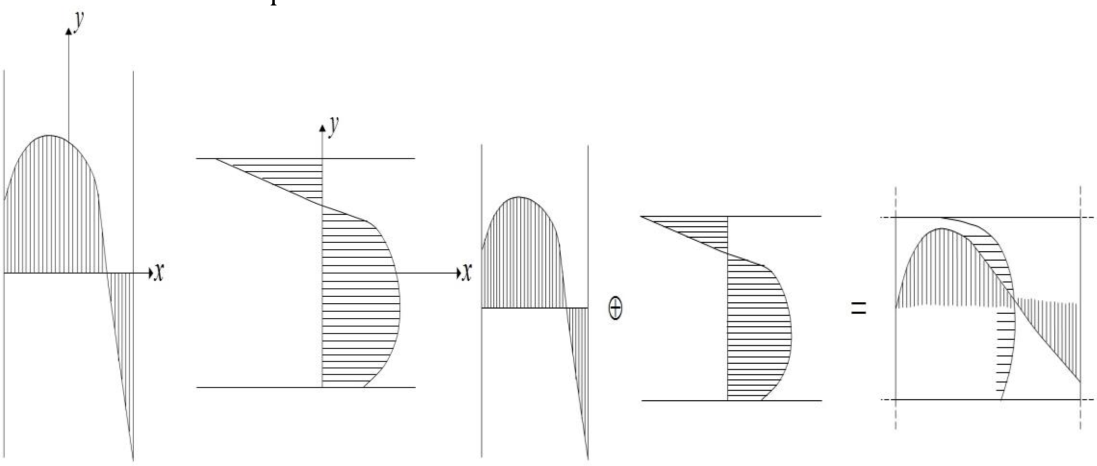
The flow in a rectangular channel with bounds that move both longitudinally and transversally has three velocity components. If the fluid is Newtonian, then all these components depend on two transverse coordinates only. If equation of the rheological state includes pressure, than the third – longitudinal – coordinate is added, and the flow itself has the highest complexity level.



(a) **(b)**

Fig. 2. Transverse fluid flow: (a) – in flat channels; (b) – in the rectangular channel

When a connection between bounds count, the type of boundary velocities and rheological characteristics are established, then the method of building the velocity field of two- and three-dimensional flows on the velocity field of the flow with lower dimension can be suggested. This method involves the representation of a transverse flow in a rectangular channel as the superposition of two transverse flows in two flat channels which are perpendicular to each other and have zero consumption. This method can be applied to both Newtonian and non-Newtonian fluids. This superposition is represented in Fig. 3.



(a) **(b)**

Fig. 3. Fluid flow in flat channels:
(a) – velocity profiles in transverse flows; (b) – superpositions of transverse flows

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The superposition lies in the fact that for each flat channel with bound that are perpendicular to each other, the longitudinal flow is considered. The equations of this flow contains terms related to another channel. The easiest way to see this is to consider the transverse flow of Newtonian fluid. Suppose there is a transverse flow on the 0y axis direction which depends on x coordinate (Fig. 4.).

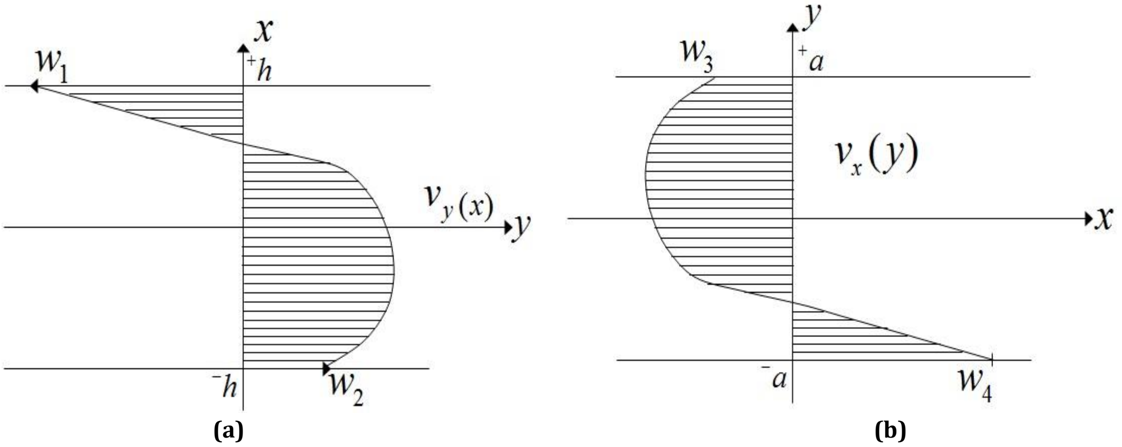


Fig. 4. Velocity profile and boundary velocities:
(a) – in longitudinal flow which depends on *x* coordinate; (b) – in longitudinal flow which depends on *y* coordinate

In this case the equations for stress balance have the following form:

$$\frac{\partial P}{\partial y} = \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y},\ v_{y(+h)} = W_1, \qquad (2)$$
$$v_y = v_y(x),\ v_{y(-h)} = W_2,\ v_y(-h) = W_2.$$

where h – if the half of flat channel width, W_1, W_2 – are the channel boundaries velocities.

In order to solve the problem (1) the connection between τ_{yx} and τ_{yy} should be specified. This can be done in several different ways, however the connection between stresses and velocities of deformations in the $\tau_{ik} = \mu\dot{\varepsilon}_{ik}$ form for Newtonian fluid should be known. Thus knowing the boundary conditions the derivatives with respect to x coordinate can always be expressed in terms of derivatives with respect to y coordinate. The derivatives with respect to x coordinate are related to the flow in channel with sides that are perpendicular to the channel from problem (2). This can be done as follows: $\frac{\partial v_y}{\partial x} \sim (W_1 - W_2)/2h$; $\frac{\partial v_x}{\partial y} \sim (W_3 - W_4)/2a$ in case when $W_{1\_}W_2 \neq 0$, $W_3 - W_4 \neq 0$. Otherwise the estimates of the following form should be used: $\frac{\partial v_y}{\partial x} \sim (v_m - W_1)/\Delta_x^+$; $(v_m - W_2)\Delta_x^-$; $\frac{\partial v_x}{\partial y} \sim (v_m - W_3)/\Delta_y^+$; $(v_m - W_4)\Delta_y^-$, where $\Delta_x^\pm$, $\Delta_y^\pm$ characterize the extremum position of velocity of the respective longitudinal flow: $\Delta_y^+ + \Delta_y^- = 2h$; $\Delta_x^+ + \Delta_x^- = 2a$. In the first case the estimates lead to $\frac{\partial \tau_{yy}}{\partial y}$ expressed in terms of $\frac{\partial \tau_{yx}}{\partial x}$. In the second case values v_m and $\Delta_x^\pm$ $\Delta_y^\pm$ act as unknown parameters which are determined after solution of the problem. In both cases the problem (2) is reduced to the longitudinal problem with one transverse coordinate. Then the same problem, but for the flat channel which is perpendicular to the first one is considered. This problem can be represented in the following form:

$$\frac{\partial P}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xx}}{\partial x},\ v_x(+a) = W_3, \qquad (3)$$
$$v_x = v_x(y),\ v_x(-a) = W_4.$$

The solution to this problem has the same form as the solution of problem (2). The estimates for velocity derivatives allow expressing $\frac{\partial \tau_{xx}}{\partial x}$ in terms of $\frac{\partial \tau_{xy}}{\partial y}$. The solution (2) and (3) should consider the zero-consumption condition. This condition leads to the equations for $\frac{\partial P}{\partial y}$ and $\frac{\partial P}{\partial x}$ in a way that values of these pressure become dependant on $W_1 - W_2\pi$ and $W_3 - W_4$.

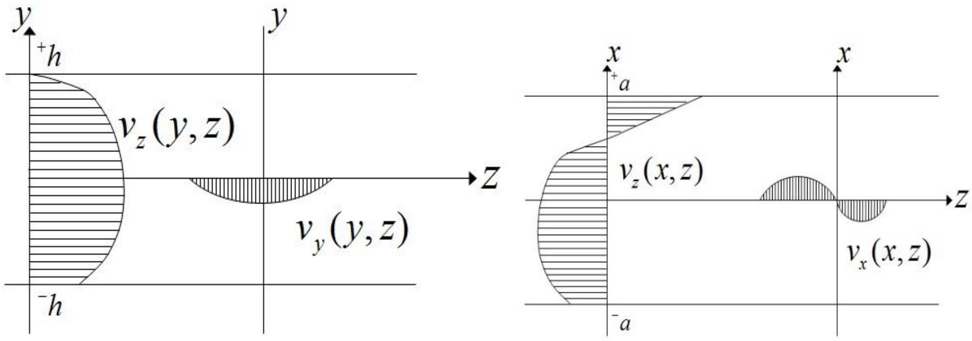
Applying the method described below to non-Newtonian fluid does not lead to any fundamental changes but make the solution for problems (2) and (3) core complicated. Here the following cases are possible: viscosity depends on the second invariant of the strain velocity tensor, or viscosity depends on pressure.

In the first case, all terms of the second invariant should be expressed in terms of corresponding derivative of the longitudinal velocity: for problem (2) it is $\frac{\partial \upsilon_y}{\partial x}$; for problem (3) – it is $\frac{\partial \upsilon_x}{\partial y}$. In the second case the longitudinal flows $\upsilon_y(x)$ and $\upsilon_x(y)$ obtain additional component $\upsilon_x(x)$ – for problem (2), and $\upsilon_y(y)$ – for problem (3). Hence, in this case the problem for longitudinal flow with one transverse component should be considered. This problem is based on the following equations:

$$\frac{\partial P}{\partial z} = \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}, \tag{4}$$

$$\upsilon_z = \upsilon_z(y,z),\ \upsilon_z(+h,z) = W_5,\ \upsilon_x = \upsilon_x(y,z),\ \upsilon_z(-h,z) = W_6.$$

Where W_5, W_6 – values for longitudinal velocities of the bounds (Fig. 5).



(a) **(b)**

Fig. 5. Longitudinal flow with transverse component for the fluid, properties of which depend on pressure: (a) – transverse component is directed along *ox* axis; (b) – transverse component is directed along *oy* axis

The solution for this problem is based in reducing $\frac{\partial \tau_{zz}}{\partial z}$ to $\frac{\partial \tau_{zy}}{\partial y}$ with the aid of described estimates. The solution for problem (4) describes longitudinal flow along the channel axis with the following boundary conditions:

$$P(z=L) = P_L,\ P(z=0) = P_0, \tag{5}$$

where L – channel length, m; P_0 and P_L – pressure values on the channel bounds, Pa.

If the zero-consumption condition is taken into account then the solution for the problem describes the transverse flows in channel which are perpendicular to each other and have two flat bound.

The problem for longitudinal flow for another pair of the bounds looks similar to the problem (4):

$$-\frac{\partial P}{\partial z} = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = 0, \tag{6}$$

$$\upsilon_z = \upsilon_z(x,z),\ \upsilon_z(+a,z) = W_7,\ \upsilon_y = \upsilon_x(x,z),\ \upsilon_z(-a,z) = W_8.$$

where W_7 and W_8 – longitudinal velocities, m (Fig. 5).

Problems (2), (3), (4), (5) lead to velocity fields which consist of two velocity fields in the intersection of flat channels. Thus, the question of choosing one or another field arises. This can be done in two ways. The first method: the velocity fields obtained as the solutions for different problems are attributed to the bounds for which they are obtained. Herewith, four fields are obtained for longitudinal velocity: two for each pair of bounds. The condition for continuity of

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longitudinal velocity values, which are calculated based on two different expressions, leads to four equations for four lines which divide the rectangle of channel cross-section into sub-areas, each of which is described by its own expression for longitudinal velocity (Fig. 6a).

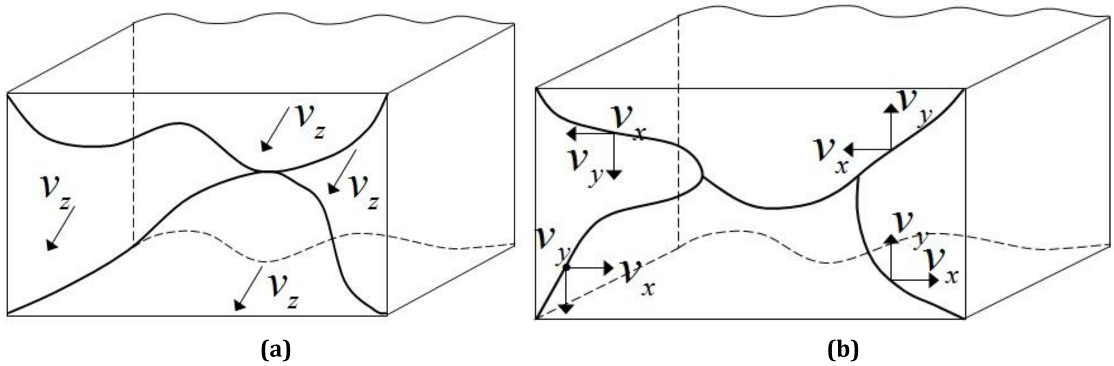

Fig. 6. Partition of cross-section of rectangular channel into sub-areas:
(a) – for longitudinal flow; (б) – for transverse flow

The same procedure is applied for transverse flow. In this case, there are four sub-areas as well. However, the condition for their fixation is not the condition for velocity continuity but the condition for continuity of the absolute velocity. Velocity vectors are tested on the lines which separate one area from another by rotating by the angle of $\frac{\pi}{2}$ (Fig. 6b). This method lies in the fact that for the rectangular channel with different sides *a* and *h* the main velocity field is the field which has longer bounds. This field is corrected by the multipliers in order to match boundary conditions for the second pair of bounds. Also, the experimentally tested statement about the fact that the influence of the pair of bounds extends inside the flow region for a distance approximately equal to the bounds length is used. Thus, for large values of this length, the influence of shorter pair of bounds weakens. In order to illustrate this statement, the channel in which $a >> h$ it can be considered. In this case, the cross-section has two sub-areas which abut the sides with length *h* and extend deep for a distance of *h*, in which expression for velocity flow in a flat channel with the width of 2*h* should be corrected in the abovementioned sense. Outside of these two regions, the flow is the same as without these two bounds. Applying this method to the transverse flow leads to these two velocity fields, each obtained for its own pair of bounds, are attributed to the entire area of rectangular cross-section, but corrected by multipliers which consider the missing pair of bounds. The method described above is more precise but also more complicated. The complexity of its application is in the fact that bounds influence extends inside the flow region for a distance of the bound area and is valid for Newtonian fluids only. This rule is also valid for non-Newtonian fluid, but the bound length should be multiplied by some multiplier which depends on the parameters of the rheological state equation.

Experimental part

In order to visualize the presented method of superposition, in this article we provide a calculation to confirm the obtained theoretical solutions by comparing them with experimental data and, based on them, assess the adequacy of the obtained models. As an example, consider the definition of such an important macrokinetic characteristic as the flow rate *V*, which is one of the most important in the flow of non-Newtonian material [24]. The calculation was carried out for a model Bingham flow in a worm machine with geometric dimensions: the length of the helical line with the step t_B = 0,086 m and the depth of the channel 2*h* = 0,007 m.

Based on the choice of geometric dimensions of the worm channel, this channel can be considered close to flat. In this case, the influence of the second pair of walls can be neglected. Whence, the expressions for determining the coordinates of the borders have the following form:

$$\gamma^{\pm} = \delta\gamma \pm \gamma; \qquad \gamma = \frac{\tau_0}{\frac{dP}{d\zeta}}; \qquad \delta\gamma = \frac{\mu(W^{+} - W^{-})}{2h\tau_0} \tag{7}$$

Substituting these expressions into the formula to determine the cost

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$$\dot{V}=\left(W_{y}^{+}+W_{y}^{-}\right)h-\left(W_{y}^{+}-W_{y}^{-}\right)y^{*}+\frac{\alpha}{2\beta}+\left(h^{2}+y^{*2}\right)+\frac{8}{15}\left(\frac{\beta}{dP/dz}\right)^{2}\cdot\left[\left(\frac{\alpha^{2}}{4\beta^{2}}+\frac{h}{\beta}\cdot\frac{dP}{dz}\right)^{5/2}\right]-$$

$$-\frac{2\beta}{3dP/dz}\cdot\left[2h\left(\frac{\alpha^{2}}{4\beta^{2}}+\frac{h}{\beta}\cdot\frac{dP}{dz}\right)^{3/2}+3\left(y^{*}\right)^{2}\cdot\left(\frac{\alpha^{2}}{4\beta^{2}}+\frac{h}{\beta}\cdot\frac{dP}{dz}\right)^{1/2}\right]. \tag{8}$$

leads to its following expression:

$$\dot{V}_1=(W^{+}+W^{-})h-\left(W^{+}-W^{-}\right)h\delta\gamma_1-$$
$$-\frac{2}{3}\frac{h^2}{\mu_0}\frac{dP_1}{d\zeta_1}\left(1-\frac{3}{2}\gamma_1+\frac{1}{2}\gamma_1^3-\frac{3}{2}\gamma_1\left(\delta\gamma_1\right)^2-3\left(\delta\gamma_1\right)^2\right). \tag{9}$$

In the case of a helical channel in a coordinate system rotating with the worm, the value of the velocity W_1 should be considered equal to zero. For the value $\gamma^{\pm}$, the following expression is a good approximation:

$$\gamma_1\approx\frac{2h\tau_0}{3\mu_0W_1^{+}}\times\left(1+\frac{2}{3}\times\frac{3\pi r_0^3}{16a_1h_1^2}\xi_s\right) \tag{10}$$

The characteristics of the flat channel were determined using the following ratios:

$$L_1=n\times\sqrt{t_B^2+\pi^2D^2},\quad tg\varphi_B={t_B}/{\pi D}\qquad a_1=t_B\cos\varphi_B \tag{11}$$

where t_B – worm step, m; n – number of turns; φ_B– the angle of elevation of the worm feather, degrees.

Substitution of the results of the calculations according to the above formulas leads to the following expressions for determining the consumption of the worm device:

$$\dot{V}=33{,}2\times N\left\{1+\frac{0{,}63\gamma}{\left(1-\gamma\right)^2}\right\}\times\frac{0{,}60\left(1-\gamma/7{,}14\right)}{1+0{,}60\times\left(1-\gamma/7{,}14\right)/\left(1-\gamma\right)},\ \gamma=\frac{1{,}31}{\mu_0W^{+}/h\tau_0}, \tag{12}$$

The obtained formulas are written in such a form and with such multipliers that the value $\dot{V}$ is calculated sm^3/s in order to simplify the comparison with experimental data.

Estimated values obtained when using a worm with a step t_B = 0.086 m and the depth of the channel $2h$ = 0.007 m and rotation frequency up to 240 spin for a second were compared with experimental data, the results are shown in fig. 7.

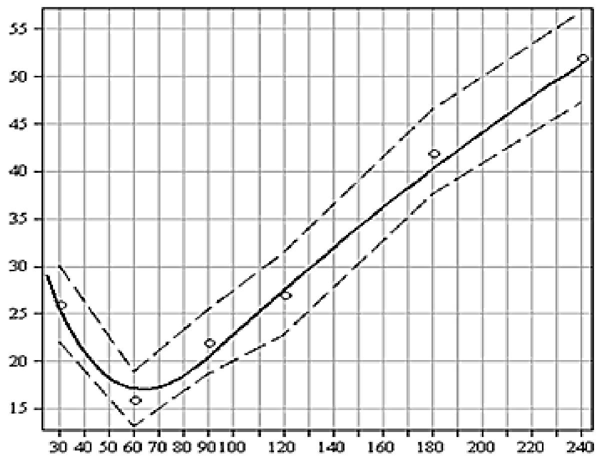

Fig. 7. Comparison of experimental data with calculated values of *V*: solid line – calculated value, o – experimental data

In this way, the obtained calculation results confirm the adequacy of the given theoretical dependencies to the real conditions of technological processes.

Conclusion

The method for reducing problems of flows with higher dimension to the problems of flows with lower dimension described in this paper can be applied to a wide variety of non-linear fluids

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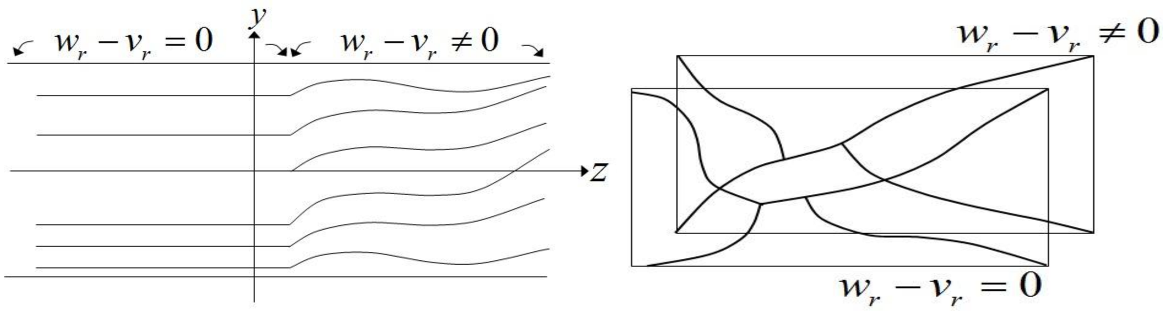
with various boundary conditions which are based not only on adhesion. This method can be extended to the flows with slipping and to the non-isothermal flows. Herein one should consider the fact, that the flow of fluids with high viscosity is accompanied by significant dissipative heat release which is described by a distributed source. The presence of slipping apart from the mentioned source indicates the necessity for accounting of surface source which is localized on the flow region bound. Sliding contact on the border is similar to the contact of two solid surfaces. Heat release in this contact depends both on pressure normal to the contact and on the magnitude of the slip. In the first case, the heat release occurs on the Coulomb type, while in the second case – on the hydrodynamic type. The task for the future is to extend method of solving three-dimensional problems to the problems with surface heat sources.

Having the possibility of non-Newtonian fluid to slide at the region bounds allows dividing all flows into two groups. The first group includes flows throughout which the first sliding conditions are complied. The second group includes flows which are partially formed by adhesion conditions and partially – by sliding conditions. For the flows of the last group, the number of velocity vector components and the number of coordinates are changed in the cross-section of the channel which has longitudinal coordinate that matches the coordinate along which the change of boundary condition form may occur [24]. For such flows the problem of “bonding” several flows of two different types should be solved. Such “bonding” should be subordinated to the conditions of continuity of all velocity and pressure components. Here, the first derivatives of the velocity in the coordinates will experience a leap. Considering connections between the components of stress tensor and strain velocities the leap of velocity derivatives means the leap of components of stress tensors. Thus, the imposition of velocity components continuity conditions is not fully consistent, because it leads to the leap of stress components and the continuity of pressure (Fig. 8).



(a) (b)

Fig. 8. Flow of the fluid with combined bounds: (a) – “bonding” lines for the fluid with sliding and adhesion conditions at the part of the bounds; (b) – partition of channel; cross-section into sub-areas when one of the sections belongs to the area with sliding and the other section belongs to the area with adhesion

The more consistent is the extension of continuity conditions on the partial derivatives of the velocity vector in cross-section, where flows with different dimensions are linked. Everything said above applies to the longitudinal components of the velocity field and to the transverse components, providing that the partition of channel cross-section rectangle into sub-areas from different sides of the cross-section of transition from one boundary conditions to another is the same. In fact, it is not so, thus the problem of “bonding” and partitioning arises. This problem requires additional study. Therefore, the method for building three-dimensional velocity and pressure fields described in this paper has certain potential of development and extension on the flows which appear during the description of large amount of practical situations in food technological processes. The method described in this paper was applied to isothermal flows without sliding for the three-dimensional problems of the flow of Newtonian, power-law, generalized and Bingham fluids in the rectangular channel with arbitrary piecewise constant distribution of bound velocities [25–26]. Herein, in some cases, it was possible to consider fluid compressibility and the dependence of the parameters of the rheological state equation of the pressure.

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